



Motion in a Circle

Name: _____ () Class: 3 / ____

Motion in a circle

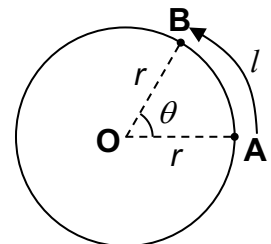
- Centripetal acceleration
- Centripetal force

Learning Outcomes

- express angular displacement in radians
- show an understanding of and use the concept of angular speed to solve problems
- recall and use $v = r\omega$ to solve problems
- describe qualitatively motion in a curved path due to a perpendicular force, and understand the centripetal acceleration in the case of uniform motion in a circle
- recall and use centripetal acceleration $a = r\omega^2$, and $a = \frac{v^2}{r}$ to solve problems
- recall and use centripetal force $F = mr\omega^2$, and $F = mv^2/r$ to solve problems.

1 Angular displacement θ

- Angular displacement is the angle swept out by the radius joining the body to the centre of the circle
- In circular motion, it is convenient to measure angles in **radians** rather than in degrees.
- Angle in radians $\theta = \frac{l}{r}$ where l : length of arc and r : radius of circle
- length of arc $l = r\theta$



2 Angular velocity ω

- Angular speed is the angle swept out by the radius per second.
- $\omega = \frac{\theta}{t}$ units: rad s^{-1} (*radians per second*)
- Angular velocity is the angular speed in a given direction. It is defined as the rate of change of angular displacement.
- In one revolution, $\theta = 2\pi$, $t = T$ where T : period
- $\omega = \frac{\theta}{t} = \frac{2\pi}{T} = 2\pi f$ where f : frequency

3 Linear speed v

- $v = \frac{l}{t}$ units: m s^{-1}
- From earlier equations, $v = \frac{l}{t} = \frac{r\theta}{t} = r\omega$
- Hence

$$v = r\omega$$

This formula would be given in ADVANCED PHYSICS exams.

Example 1

Calculate the angular speed (i) in degrees per second, (ii) in radians per second of

- (a) a fan blade rotating at 2.5 r.p.m. (*revolutions per minute*)
- (b) the minute hand of a clock.

[(a)(i) 15° s^{-1} , (ii) 0.26 rad s^{-1} , (b)(i) $0.10^\circ \text{ s}^{-1}$, (ii) $1.7 \times 10^{-3} \text{ rad s}^{-1}$]

3 Centripetal acceleration a & centripetal force F

- Consider a particle going around a circle of radius r with a constant speed v .
- Although the speed is constant, the **direction of the velocity is always changing**, so there is **acceleration**.
- This acceleration is directed **towards the centre of the circle**. And is given by:

$$a = \frac{v^2}{r}$$

This formula would be given in ADVANCED PHYSICS exams.

- This means that the **net force** on the particle must be **directed towards the centre of the circle**.

$$F_{\text{net}} = ma = m \frac{v^2}{r} \quad \text{or} \quad F_{\text{net}} = ma = m r \omega^2$$

- We call this net force **centripetal force** and this provides the **centripetal acceleration** of the particle.

Example 2

Calculate the acceleration of a car which is moving in a circle of radius 90 m and has a constant speed of

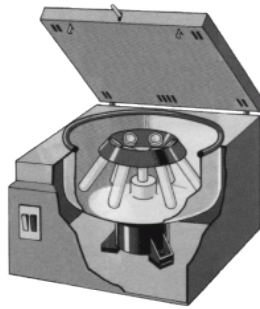
(a) 10 m s^{-1}

(b) 20 m s^{-1}

(a) 1.1 m s^{-2} , (b) 4.4 m s^{-2}

Example 3

A centrifuge is commonly found in a laboratory in which solid or liquid particles of different densities are separated by rotating them in a tube in a horizontal circle.



From http://www.ktf-split.hr/glossary/en_o.php?def=centrifuge

A centrifuge is required to give an acceleration of $1000g$ to a particle at a distance of 8.5 cm from the axis of rotation. Find the necessary angular speed of the centrifuge.

[340 rad s^{-1}]

Examples of circular motion & centripetal force

- A car making a circular bend or turn – the centripetal force is provided by friction between the road and the tyres of the car.
- The Earth revolving around the Sun – the centripetal force is provided by the gravitational attraction force exerted on the Earth by the Sun.

Example 4

A geosynchronous satellite makes a complete circular orbit around Earth once every 24 hours. The radius of the orbit is 4.2×10^7 m.

(a) What is its angular speed?

(b) What is its speed?

(c) What is its centripetal acceleration?

[(a) $7.3 \times 10^{-5} \text{ rad s}^{-1}$, (b) 3.1 km s^{-1} , (c) 0.22 m s^{-2}]

4 General Problem-Solving Steps in a Circular Motion Question.

1. Draw a free body diagram representing all the forces acting on the object.
2. Locate the centre of the circular motion, and determine the forces or components of forces **directed to that centre**.
Write "[force(s)] provide for the centripetal force"
3. Apply Newton's 2nd law of motion ($F_{\text{net}} = ma$):

$$F_{\text{net}} = m \frac{v^2}{r} \quad \text{or} \quad F_{\text{net}} = m r \omega^2$$

where F_{net} is the **resultant** of the individual forces, or components of forces, that are **directed to the centre of the circle**.

4. Manipulate the equation and solve for unknowns required.

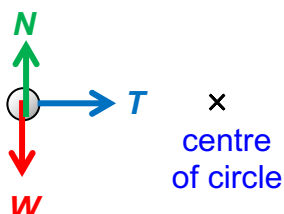
Worked example 1

The body of mass 0.20 kg moves in uniform horizontal circular motion with a speed of 2.0 m s^{-1} . The string has a length of 0.25 m.

Calculate the tension in the string.

Solution:

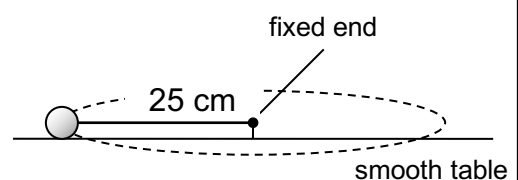
Free body diagram:



Tension provides for the centripetal force

Applying $F_{\text{net}} = ma$,

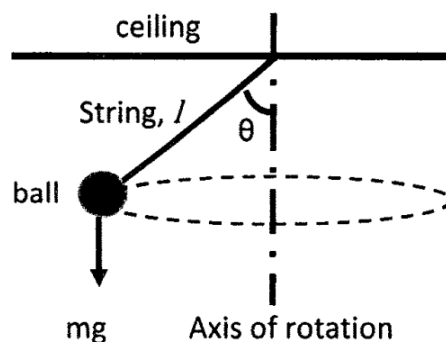
$$\begin{aligned} T &= m \frac{v^2}{r} \\ &= 0.20 \times \frac{2.0^2}{0.25} = 3.2 \text{ N} \end{aligned}$$



Worked example 2 (SJPO 2016 Q14)

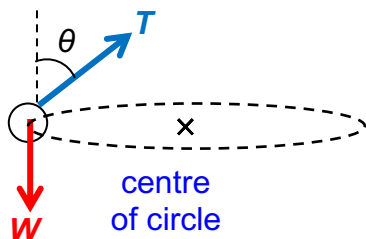
A ball with mass m is hung from the ceiling with a massless string of length l as shown in the diagram. It moves in uniform circular motion with an angular velocity ω .

What is the magnitude of the tension in the string?



Solution:

Free body diagram:



The **horizontal component of tension** provides for the centripetal force

$$F_{net} = ma$$

$$T \sin \theta = m r \omega^2$$

$$T \sin \theta = m (l \sin \theta) \omega^2$$

$$T = m l \omega^2$$

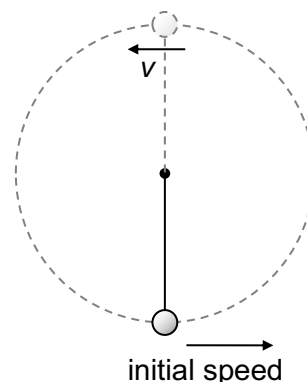
Note:

The vertical component of tension balances the weight (F_{net} in the vertical direction = 0)

Example 5

A body of mass 0.60 kg is hanging on a string of length 1.4 m is given an initial horizontal speed and subsequently, it moves in a vertical circular motion.

(a) Calculate the tension in the string when the body is at the top if its speed at the top is 5.0 m s^{-1} .

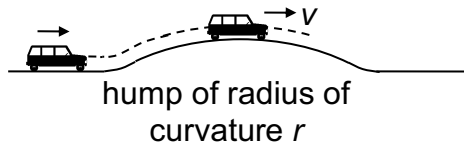


(b) Calculate the minimum speed of the body at the top if the string is to remain taut.

[(a) 4.7 N, (b) 3.7 m s^{-1}]

Example 6

A car travels over a hump with radius of curvature $r = 20$ m.



Calculate the maximum speed of the car if the car is to remain in contact with the road.

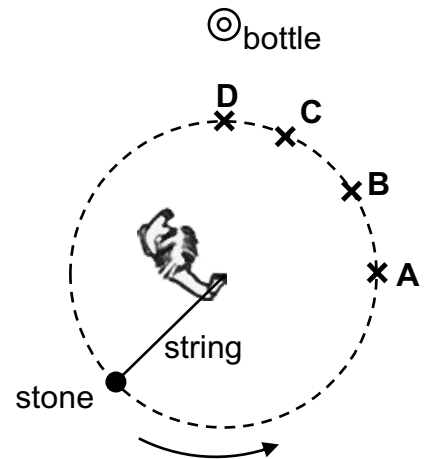
[14 m s⁻¹]

Exercises

1. A child swings a stone on a string in a horizontal circle, and releases it to strike a bottle.

At which point should she release the stone to strike the bottle?

Reasoning:

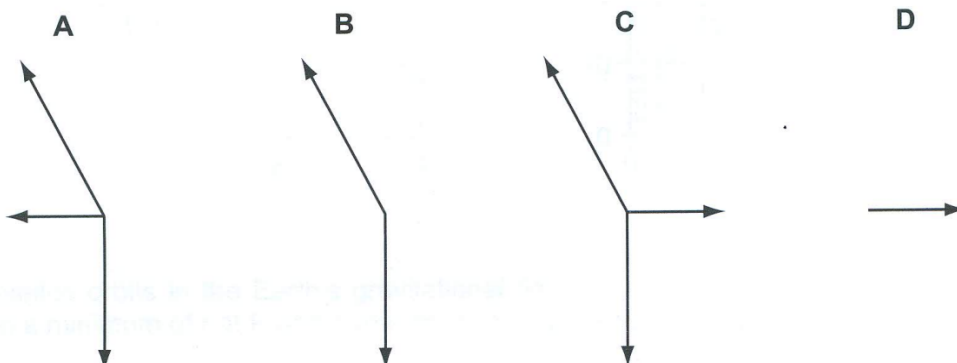
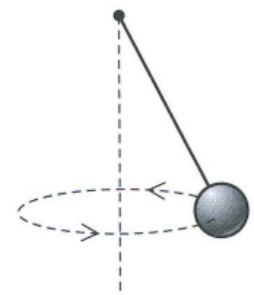


H2 Physics 2013 Q11

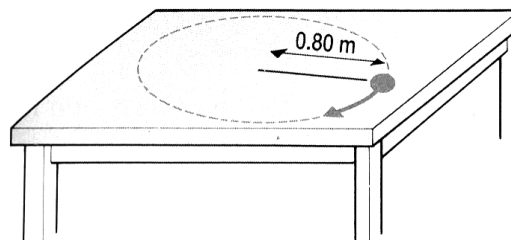
2. A small ball suspended from a light thread moves in a horizontal circle at a constant speed.

A student draws the forces acting on the ball but fails to label them.

Which diagram shows the correct forces acting on the ball?



3. The figure shows a mass of 0.40 kg rotating in a circular path of radius 0.80 m on a friction-free table. It is attached by a string to a peg at the centre of the circle.



(a) Draw a free body diagram for the mass.

(b) Determine the force which the string exerts on the mass when the mass is moving at a constant speed of 3.5 m s^{-1} .

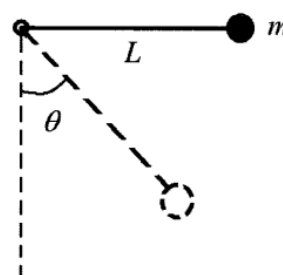
4. A rope is tied to a bucket of water, and the bucket is swung in a vertical circle of radius 1.2 m. Determine the minimum speed of the bucket at the highest point of the circle if the water is to stay in the bucket throughout the motion.

Challenge Yourself

SJPO 2022 Q 10 and 11

(hint: Q5 and Q6 make use of conservation of energy)

5. A simple pendulum, consisting of a string tied to a frictionless fixed pivot at one end and a bob of mass m at the other end, is gently released from the horizontal position as shown.



After the release, the tension of the string is given by _____.

- A** $3mg \cos\theta$ **B** $3mg \sin\theta$ **C** $mg \cos\theta$ **D** $mg \sin\theta$ **E** $4mg \cos\theta$

6. SJPO 2022 Q11

In order for the string not to break during the swing, it must be able to withstand a tension of _____.

- A** $2mg$ **B** $4mg$ **C** $8mg$ **D** $6mg$ **E** $3mg$

Answers: 1. B 2. B 3. (b) 6.1 N 4. 3.5 m s^{-1} 5. A 6. E