2026 Sec 3 Advanced Physics

Kinematics: Non-linear Motion ANSWERS

Name: ____________________ ( ) Class: 3 / ___

Kinematics

- Non-linear motion

Learning Outcomes

(a) represent a velocity vector as two perpendicular components

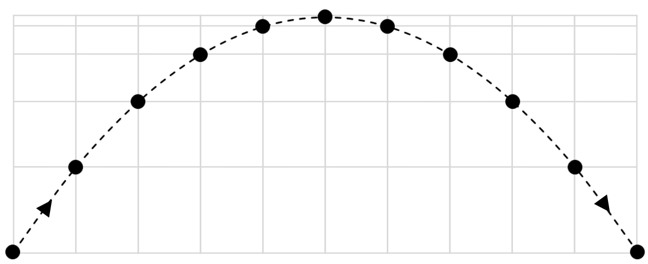
(b) describe and explain motion due to a uniform velocity in one direction and a uniform acceleration in a perpendicular direction.

References:

- See **projectile motion** at http://www.physicsclassroom.com/class/vectors
- See phET projectile motion simulation at https://phet.colorado.edu/sims/html/projectile-motion/latest/projectile-motion_en.html
- See http://hyperphysics.phy-astr.gsu.edu/hbase/index.html
  - select Mechanics -> select **Newton's Laws**: focus on **Force of gravity** -> Horizontal launches & Range of Projectile

1 Projectile motion

- A projectile is an object that moves in **two dimensions** under the influence of **gravity** and nothing else.
- If air resistance is negligible, any projectile will follow the same type of path: a trajectory with the mathematical form of a parabola, of the form $y = ax^2 + bx + c$, where a is negative.

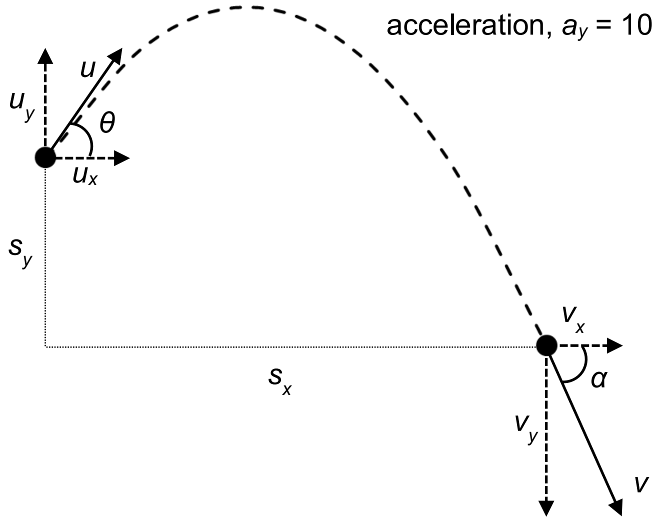


Trajectory of an object projected at an angle

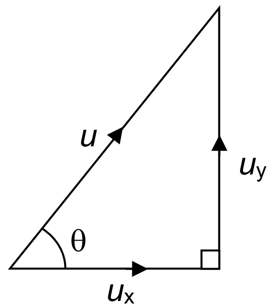
- Examples of projectile motion: balls flying through the air, long jumpers, and cars doing stunt jumps.
- To analyse the projectile motion near the Earth's surface, we often apply the following three assumptions:
  - the acceleration due to gravity, g, is constant (10 m s^{-2} pointing downwards) over the entire motion
  - there is no horizontal acceleration
  - the effect of air resistance is negligible

2 **Two perpendicular components of projectile motion**

- Consider an object that is projected with an initial velocity u directed at an angle θ as shown below.



- Considering the initial velocity:



  - the x-component of the initial velocity is $u_x = u \cos \theta$
  - the y-component of the initial velocity is $u_y = u \sin \theta$
  - $u^2 = u_x^2 + u_y^2$
  - $\tan \theta = \dfrac{u_y}{u_x}$ *(Ensure calculator in degree mode)*

- The projectile's motion can be analysed as two perpendicular components.

	horizontal component	**vertical component**
displacement *s*	s_x	s_y
acceleration *a*	$a_x = 0$ m s^{-2} no acceleration	$a_y = 10$ m s^{-2} **downwards** (if near Earth's surface) (acceleration under free-fall)
Applying equations of motion *(Indicate **sign convention**!):*		
$v = u + at$	$v_x = u_x$ constant velocity	$v_y = u_y + a_y t$ varying velocity
$s = ut + ½ at^2$	$s_x = u_x t$ s_x is the **range** of the projectile	$s_y = u_y t + \frac{1}{2} a_y t^2$
$v^2 = u^2 + 2as$	$v_x^2 = u_x^2$	$v_y^2 = u_y^2 + 2a_y s_y$

- The final speed is $v = \sqrt{v_x^2 + v_y^2}$

- $\tan \alpha = \dfrac{v_y}{v_x}$

3 Problem solving approach

(1) **Sketch** a diagram to show the complete path of a projectile and indicate given and unknown variables.

(2) Analyse the motion in two perpendicular directions **independently** using suitable symbols with suitable subscripts, e.g. x and y.

- If the initial velocity is given, **resolve it into its x and y components**.

(3) Neglecting air resistance, any projectile experiences

- no horizontal acceleration (constant horizontal velocity).
- a vertical acceleration: choose **sign convention**, apply equations of motion to vertical motion

(4) If the instantaneous velocity (or direction) of the projectile is needed, add the two components of the velocity (v_x and v_y) using **vector addition**.

Additional tips:

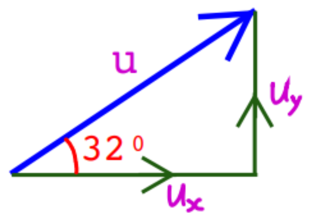
- When an object is **at its highest point of motion**, the vertical component of its velocity, $\boldsymbol{v_y}$ **is always zero**.
- If an object is **projected horizontally** with a speed u,
  - the horizontal component of initial velocity $\boldsymbol{u_x}$ **is** $\boldsymbol{u}$,
  - the vertical component of its initial velocity, $\boldsymbol{u_y}$ **is zero**.

4 Effect of air resistance on projectile motion

- reduce the maximum height
- reduce the maximum range
- make the angle of descent steeper
- distort the shape of the path away from a parabola

Example 1

A ball is thrown at an angle of 32° from the ground with a speed of 25 m s^{-1}. Calculate the magnitude of the horizontal and vertical components of its initial velocity.



horizontal component:

$u_x = u \cos\theta$ = 25 cos 32°
= 21.20
=21 m s^{-1} (2 s.f.)

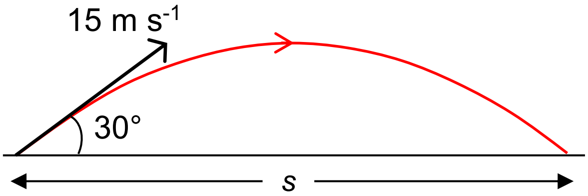
vertical component:

$u_y = u \sin\theta$ = 25 sin 32°
=13.25
=13 m s^{-1} (2 s.f.)

- *Note: Check u =*

[21 m s^{-1}, 13 m s^{-1}]

Example 2
A football is kicked on level ground at a velocity of 15 m s^{-1} at an angle of 30° to the horizontal.



(a) Determine the time taken by the football till its first bounce on the ground.

- *sign convention: upward is positive, $a = -g = -10$ m s^{-2}*
- *vertical motion:* $s_y = u_y t + \frac{1}{2} a_y t^2$

Since the ball returns to the same vertical level, $s_y = 0$

$0 = (15\sin 30°)\, t + ½\, (-10)\, t^2$

$t = 0$ s (initial) or $t = 2(15 \sin 30°)/10 = 1.5$ s

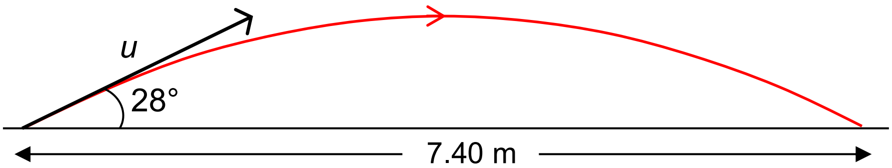
(b) Hence, calculate the range of the football.
(note: range refers to the horizontal displacement)

- *horizontal motion:*

$s_x = u_x t$
$= (15 \cos 30°)(1.5)$
$= 19$ m (2 s.f.)

[19 m]

Example 3
An athlete competing in the long jump leaves the ground at an angle of 28° and makes a jump of 7.40 m.
(a) Sketch the path of the athlete and include the information provided above.



(b) Calculate the speed at which the athlete took off.

horizontal motion: $s_x = u_x t$

$7.40 = (u \cos 28°)\, t$ …………… (1)

sign convention: upward is positive

vertical motion: $s_y = u_y t + \frac{1}{2} a_y t^2$

$0 = (u\sin 28°)\, t + ½\, (-10)\, t^2$

$t = \frac{u \sin 28°}{5}$ …………………………..(2)

Substitute equation (1) into (2)

$$7.40 = \frac{u^2(\cos 28°)(\sin 28°)}{5}$$

$$u = \sqrt{89.26}$$

$$= 9.448 \approx 9.4 \text{ m s}^{-1} (2\text{s.f.})$$

(c) If the athlete had been able to increase this speed by 5%, determine the percentage difference this would have made to the length of the jump.

From the equation used in part (b), $s_x = \dfrac{u^2(\cos 28°)(\sin 28°)}{5}$

hint: express s_x in terms of u

If the speed increases by 5%, $u_{new} = 1.05\,u$

Since the angle remains constant, $s_x \propto u^2$

$$s_{x,new} \propto u_{new}^{\ 2}$$
$$\propto (1.05\,u)^2$$
$$\propto 1.10\,u^2$$

The length of jump increases by 10 %

Alt presentation:

$$\frac{s_{x,new}}{s_x} = \left(\frac{u_{new}}{u}\right)^2$$
$$\frac{s_{x,new}}{s_x} = \left(\frac{1.05u}{u}\right)^2$$
$$s_{x,new} = 1.10 s_x$$

[(b) 9.4 m s^{-1}; (c) 10%]

Example 4

A stone is thrown horizontally from the top of a cliff of height 32 m, as shown in Fig. 4.1. Air resistance is negligible.

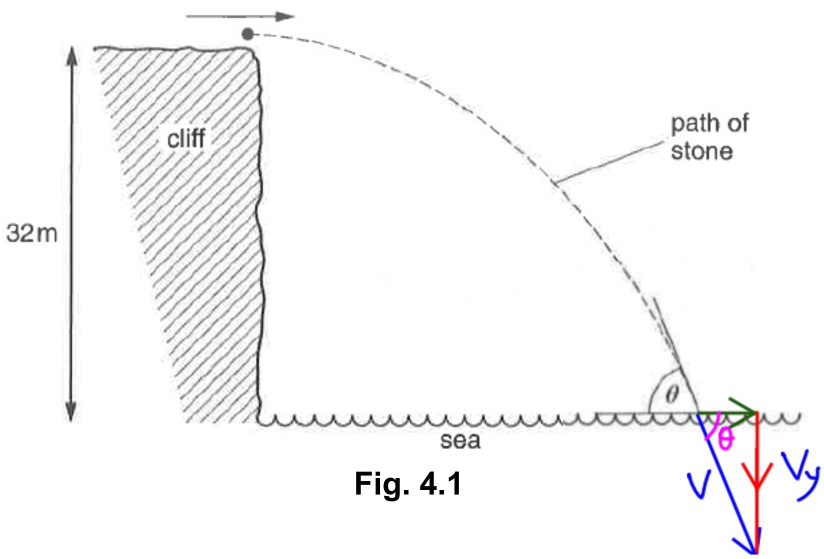
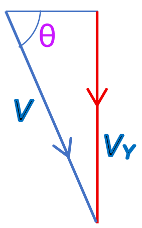


Fig. 4.1

(a) Determine the vertical component of the velocity of the stone just before it hits the sea.

note: stone is thrown horizontally, $u_y = 0$

sign convention: downward is positive



vertical motion: $v_y^{\ 2} = u_y^{\ 2} + 2a_y s_y$

v_y^2 = 0 + 2(10)(32)

v_y = 25.30

= 25 m s^{-1} (2 s.f.), downwards

(b) The stone enters the sea with a speed of 34 m s^{-1}. Determine the angle θ to the horizontal of the stone's path as it hits the sea.

The speed v is 34 m s^{-1}

$v_y = v \sin\theta$

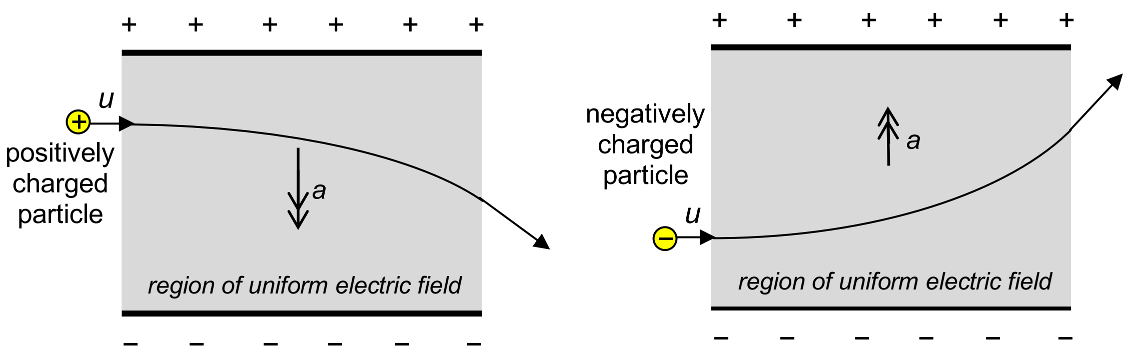
$25.30 = 34 \sin\theta$

$$\theta = \sin^{-1}\left(\frac{25.30}{34}\right) = 48°$$

[(a) 25 m s^{-1}; (b) 48°]

5 Charged particles in uniform electric field

- Two parallel plates can be setup using an e.m.f. source to become charged with opposite charges. This creates a region of **uniform electric field**.
- A charged particle will experience a **constant acceleration** in this region of uniform electric field.
- Hence, if the charged particle enters the electric field with a velocity u at right angles to the direction of the acceleration, the particle will follow a **parabolic path**.

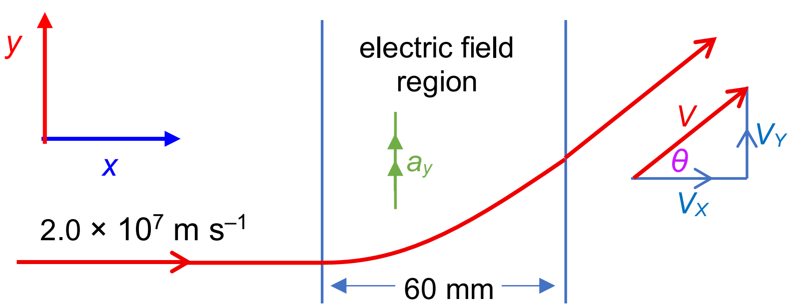


Motion of charged particles with initial velocity perpendicular to the direction of acceleration in regions of uniform electric field

Example 5

An electron, travelling with a velocity of 2.0 x 10^{7} m s^{-1} in a horizontal direction, enters a uniform electric field. This field gives the electron a constant acceleration of 5.0 x 10^{15} m s^{-2} in a direction perpendicular to its original velocity.

The electron travels a horizontal displacement of 60 mm within the region of electric field. Determine the magnitude and direction of the velocity of the electron when it leaves the field. (v and θ).



Horizontal motion: $v_x = u_x = u = 2.0 \times 10^7$ m s^{-1}

$s_x = u_x t$
$60 \times 10^{-3} = 2.0 \times 10^7\ t$
$t = 3.0 \times 10^{-9}$ s

sign convention: upward is positive

Vertical motion: $v_y = u_y + a_y t$
$= 0 + (5.0 \times 10^{15})(3.0 \times 10^{-9}) = 1.5 \times 10^7$ m s^{-1}

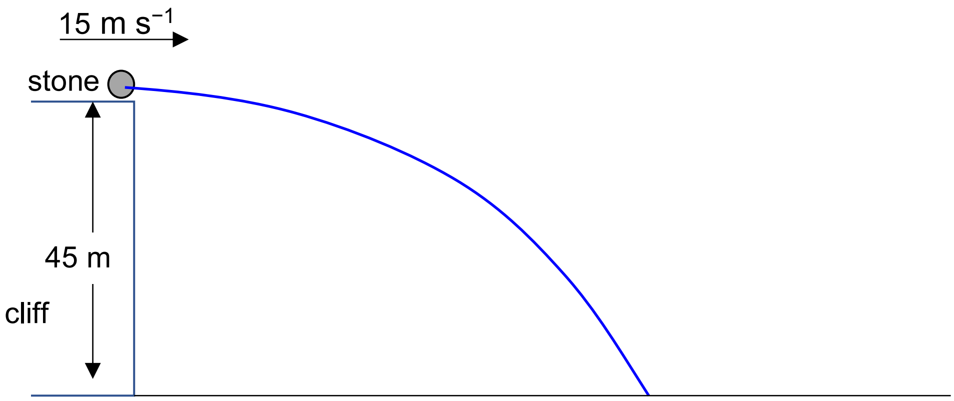
$$v = \sqrt{v_x^2 + v_y^2}$$
$$= \sqrt{(2.0\times10^7)^2 + (1.5\times10^7)^2} = 2.5\times10^7 \text{ m s}^{-1}$$

$$\tan\theta = \frac{v_y}{v_x} = \frac{1.5}{2.0}$$
$$\theta = \tan^{-1}\left(\frac{1.5}{2.0}\right) = 37°$$

[2.5×10^7 m s^{-1}; 37° to the horizontal]

Exercises

1 A stone is thrown from the top of a cliff, 45 m high above level ground, with an initial velocity of 15 m s^{-1} in a horizontal direction.
(a) Sketch the path of this stone and include the information provided above.



(b) Determine the time taken for the stone to reach the ground.

sign convention: downward is positive

vertical motion: $s_y = u_y t + \frac{1}{2} a_y t^2$

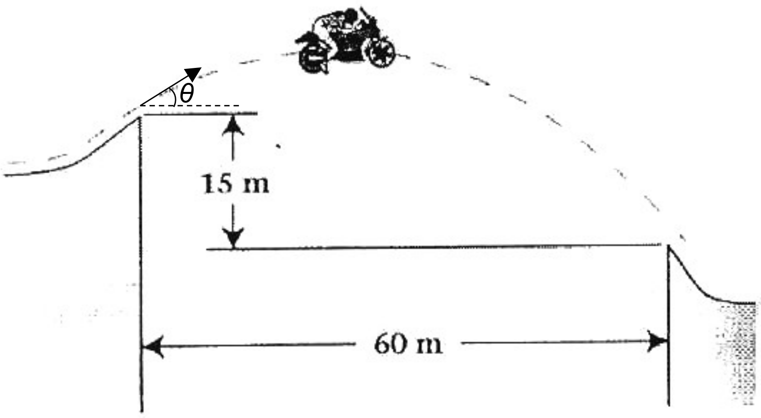
$45 = 0\ t + ½ (10)\ t^2$
$t^2 = 9.0$
$t = 3.0$ s

(c) Hence, calculate the horizontal displacement of the stone from its initial launch position when it reaches the ground.

Horizontal motion:

$s_x = u_x t$
$= 15 \times 3.0$
$= 45$ m

2 A Hollywood daredevil plans to successfully jump across the canyon shown in the figure below.



He desires a 3.0 s flight time. You may assume there is no air resistance.

(a) Determine the magnitude of the horizontal component of his velocity, u_x.

Horizontal motion:

$s_x = u_x t$

$60 = u_x (3.0)$

$u_x = 20 \text{ m s}^{-1}$

u_x = ... m s^{-1}

(b) Determine the magnitude of the vertical component of his initial velocity, u_y.

sign convention: upward is positive

vertical motion: $s_y = u_y t + \frac{1}{2} a_y t^2$

$-15 = u_y (3.0) + \frac{1}{2}(-10)(3.0)^2$

$u_y = 10 \text{ m s}^{-1}$

u_y = ... m s^{-1}

(c) Hence, determine his angle of projection θ.

$$\theta = \tan^{-1}\left(\frac{u_y}{u_x}\right) = \tan^{-1}\left(\frac{10}{20}\right) = 26.6°$$

θ = ...

(d) Determine his velocity just before landing.

horizontal motion: $v_x = u_x = 20 \text{ m s}^{-1}$

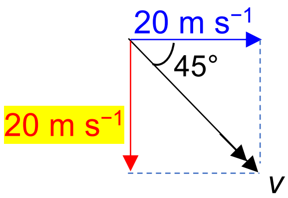
sign convention: upward is positive

vertical motion: $v_y = u_y + a_y t$

$= 10 + (-10)(3.0)$

$= -20 \text{ m s}^{-1}$

vertical component of the velocity is 20 m s^{-1} downwards



$$v = \sqrt{v_x^2 + v_y^2}$$
$$= \sqrt{20^2 + 20^2}$$
$$= 28 \text{ m s}^{-1} \text{ (2s.f.)}$$

direction: 45° clockwise below the horizontal

velocity = ... m s^{-1}

direction: ....

Answers:	1. (b) 3.0 s, (b) 45 m
	2. (a) 20 m s^{-1} (b) 10 m s^{-1} (c) 26.6°, (d) 28 m s^{-1}, 45°clockwise below the horizontal