Physics TIP

Using the system of binary stars to weigh stellar objects

Anastasia Yang, Zhao Binxi Group 11

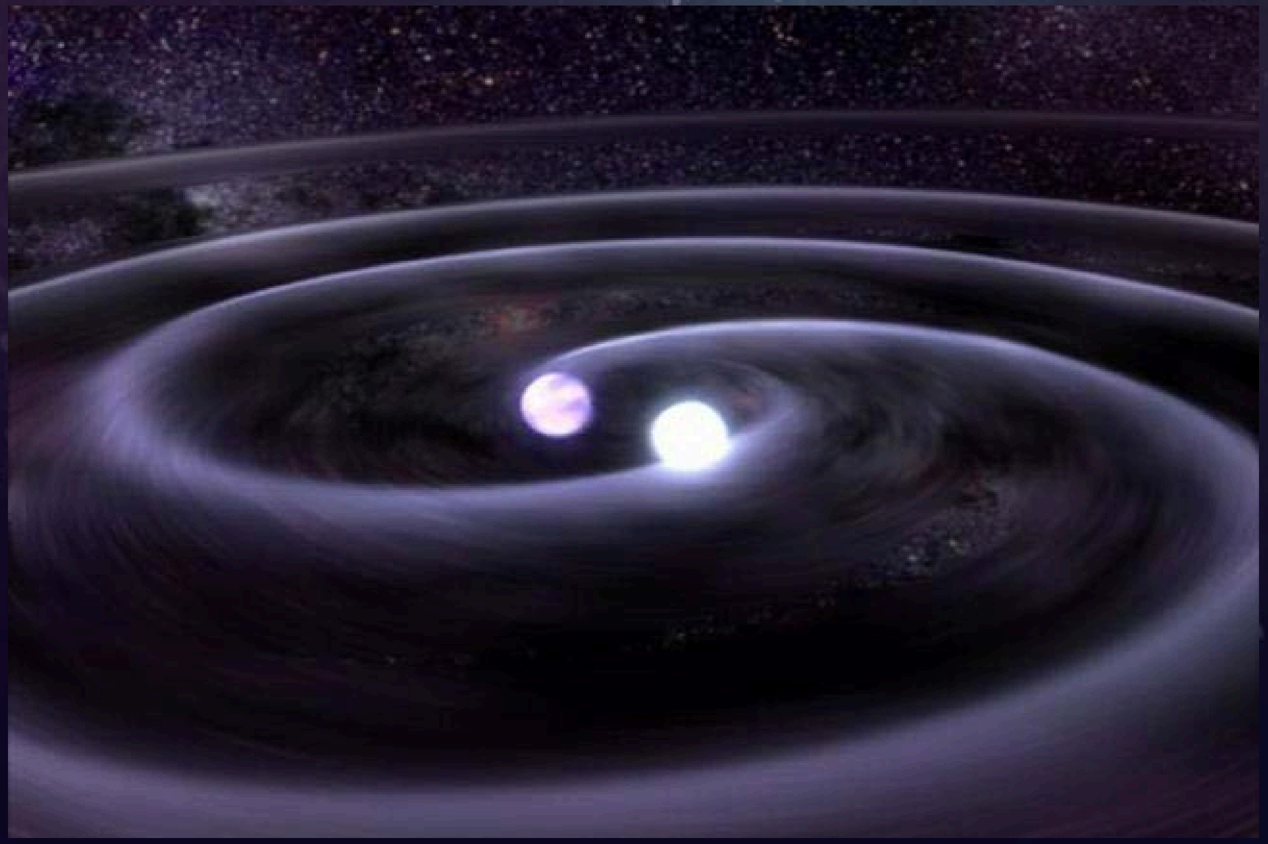
Table of *contents*

01

History of the discovery of Binary Stars

Binary Stars, What is it?

A binary star is a system of two gravitationally bound stars that orbit a common center of mass called a barycenter.



Timeline of Binary Stars

1650

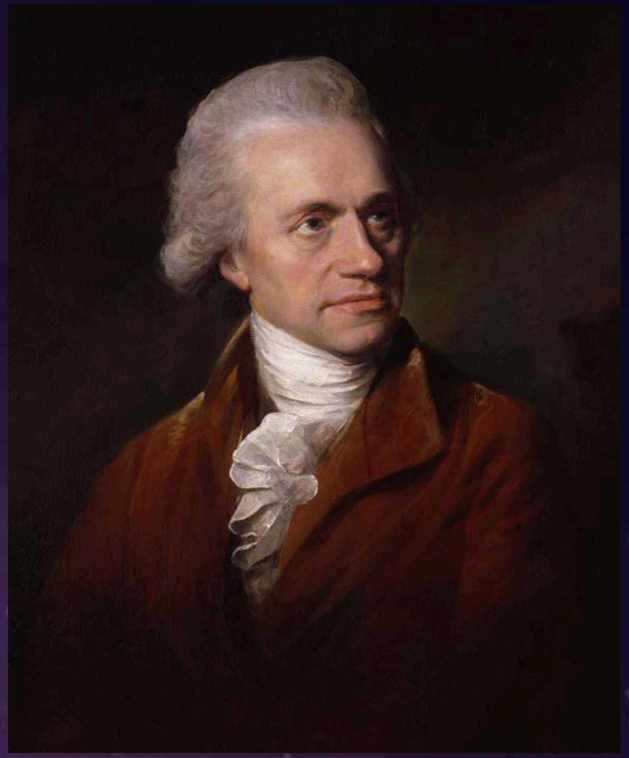
The first evidence of a double star, marked as Mizar, was discovered by Italian astronomer, Giovanni Battista Riccioli.

1767

John Michell applied statistical methods to demonstrate that the observed proximity of many double stars was too high to be a chance alignment, suggesting that gravitational attraction is in effect. Since then, astronomers have discovered thousands of binary stars.

1803

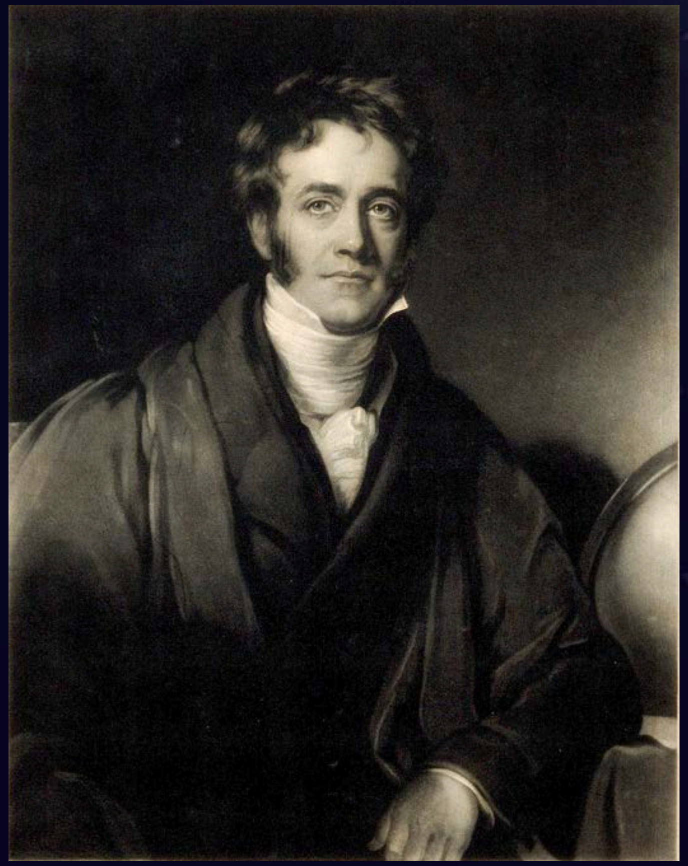
William Herschel discovered the first binary star, Castor in Gemini after He noticed that the fainter component of Castor had slightly changed its position relative to the brighter component. This was evidence that two stars were orbiting around a mass.

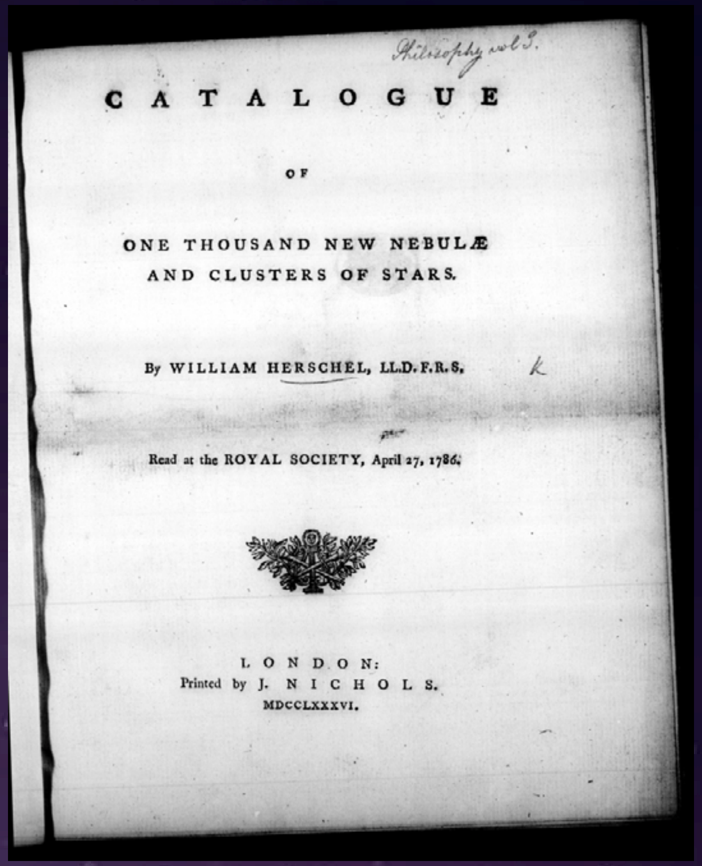


Timeline of Binary Stars

After 1816

William Herschel's son, John, continued his father's work after 1816, and with his friend James South, produced a catalogue of 380 binary stars. In 1833, he packed 3 of his father's telescopes and traveled to South Africa, where he spent the next four years observing clusters, nebulae and over 2100 binary stars.

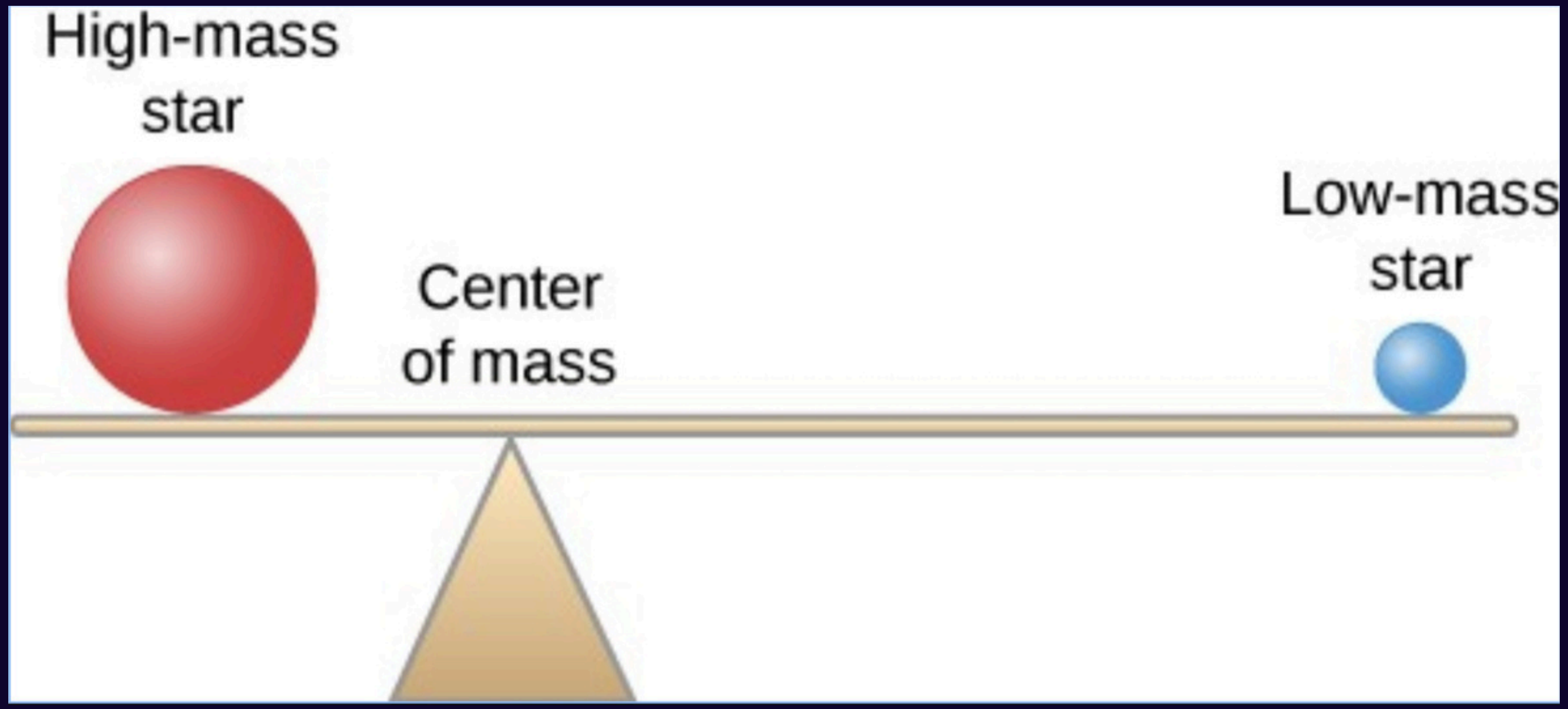


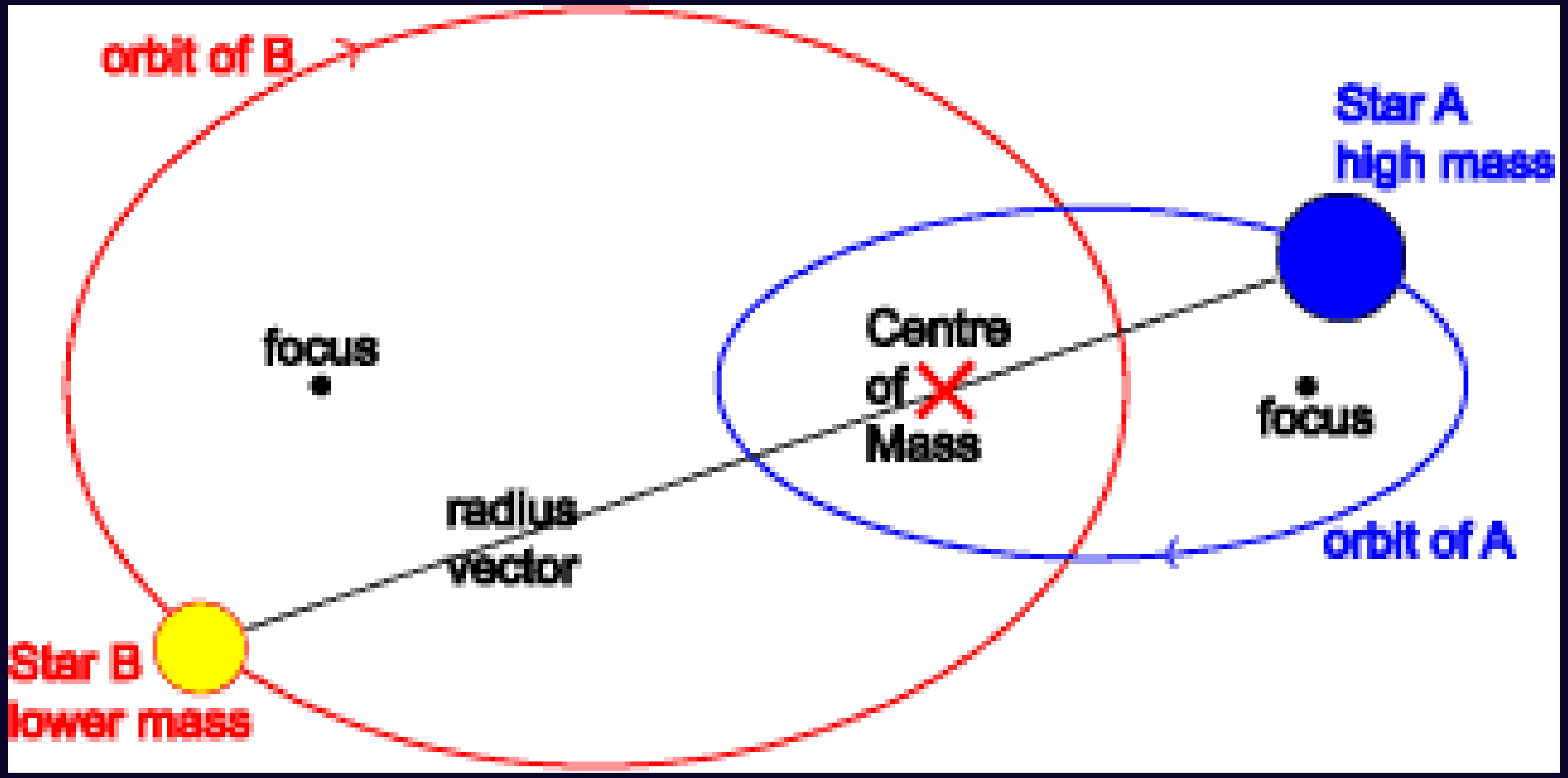


02

The Binary Star System

In a Binary Star System



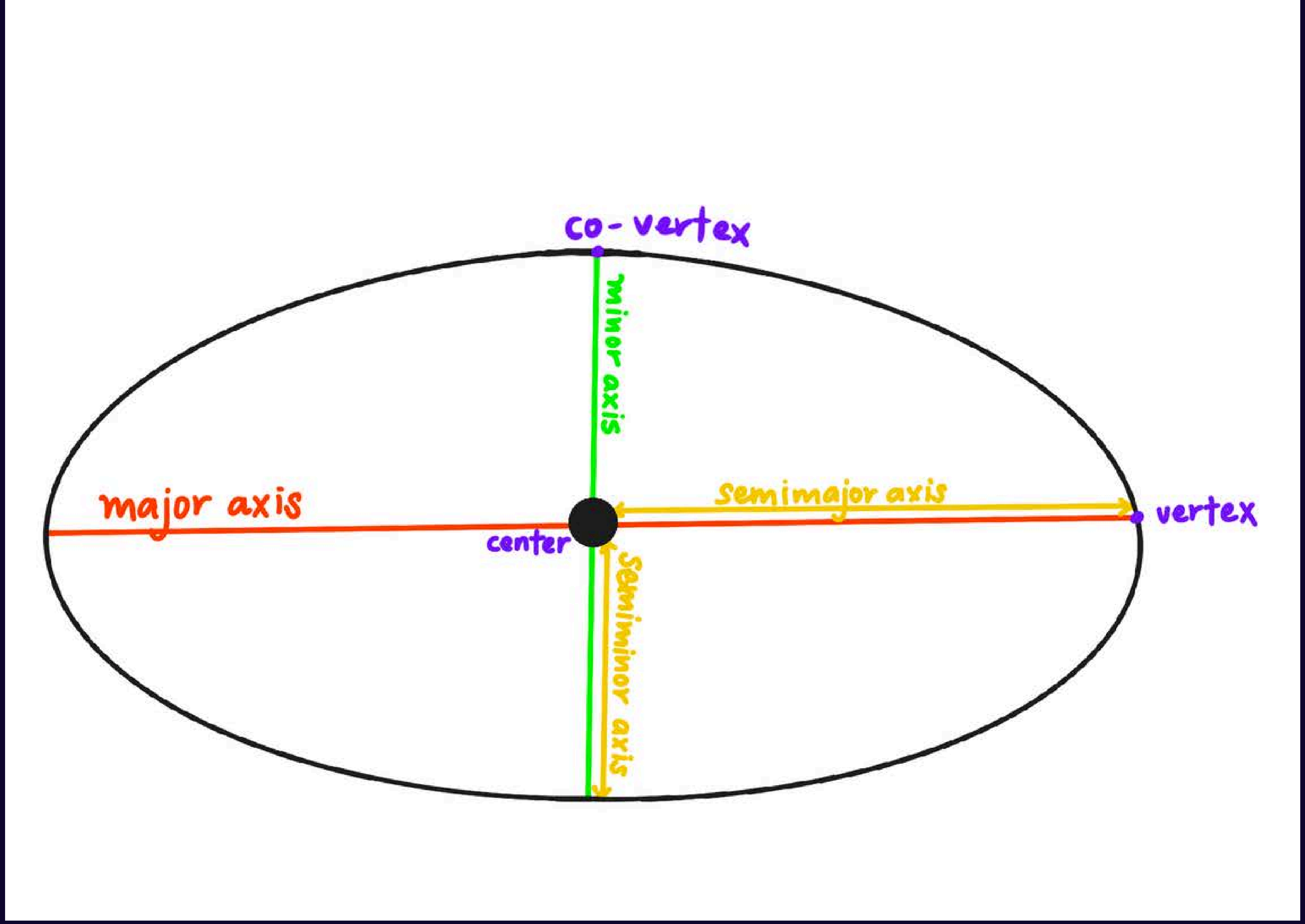


- Two gravitationally bound stars orbit a common centre of mass called a Barycenter.
- It is the weighted average of their positions. They need not have the same mass, size or brightness.
- The larger star is called the **primary star** and the smaller star is called the **secondary star**.
- The two stars behave like a see-saw, where the centre of mass is the pivot. The primary star will be found closer to the centre of mass, while the secondary star will be found farther away from it.

03

Understanding Orbits

Parts of an orbit



Center
The midpoint where the major and minor axes intersect.
Major axis
The distance between 2 end vertices. The center divides the major axis into 2 equal halves, the semi-major axes.
Minor axes
The distance between the end co-vertices. The centre divides the minor axis into 2 equal halves, the semi-minor axes.
Vertex
The extreme point that forms the major axis.
Co-vertex
The extreme point that forms the minor axis.

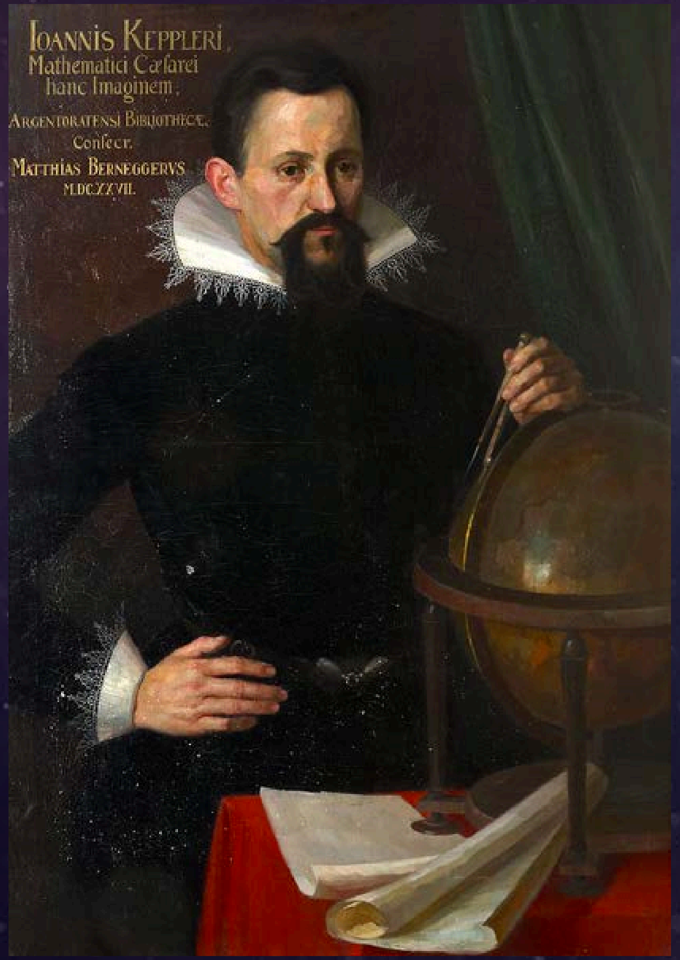
04

Johannes Kepler

Johannes Kepler

27 December 1571 (Weil der Stadt, Germany) - 15 November 1630 (Regensburg, Germany)

- German astronomer, mathematician, astrologer, natural philosopher and composer



Johannes Kepler

- Kepler began studying theology in Tübingen in 1589, he then changed to studying natural sciences instead.
- After that, he accepted various teaching positions and became the royal mathematician and court astronomer for Rudolf II of Prague in 1600
- He was apprenticed to Danish astronomer Tycho Brahe and inspired by his extensive and precise astronomical observations.
- In 1609, Kepler published his first two laws. The third and final law was published a decade later in 1619.

05

Kepler's 3rd Law and Newton's Reformulation

Kepler’s 3rd Law

- Kepler’s 3rd law: The square of the orbital period of a planet is directly proportional to the cube of the semi-major axis of its orbit.

$$P^2 = k\,a^3$$, where k is a constant.

To find the weight of stellar masses, we will need to Newton’s Reformulation of Kepler’s 3rd Law.

Combining Kepler's 3rd Law and Newton's Law of Gravitation

Newton's Law of Universal Gravitation states that every particle attracts every other particle in the universe with force directly proportional to the product of the masses and inversely proportional to the square of the distance between them.

Newton's Universal Law of Gravitation: $F = G\frac{Mm}{r^{2}}$

Where,
F is the gravitational force between bodies (N)
G is the gravitational constant (6.6710-11 N m2 kg-2)
M and **m** are the masses of each body (kg)
r is the distance between the centers of masses of the bodies (m)

Combining Kepler's 3rd Law and Newton's Law of Gravitation

Newton's reformulation of Kepler's 3rd law can be derived from Newton's universal law of gravity and circular motion as gravitational force provides for centrepetal force. By equating the gravitational to centripetal force, we get this equation.

$$F_{grav} = F_{cent}$$

In order to carry out the calculations without considering both bodies separately, i.e. turn the two body problem into a one body problem, we use reduced mass.

$$\mu = \frac{Mm}{M+m}$$

Where,
μ is the reduced mass (kg)
M and **m** are the masses of each body (kg)

Combining Kepler’s 3rd Law and Newton’s Law of Gravitation

By substituting into the centripedal force, we can get this equation,

$$F_{cent} = \mu \frac{v^2}{r}$$

Since gravitational force provides for the centripedal force,

$$G\frac{Mm}{r^2} = \mu \frac{v^2}{r}$$

Substitute in μ:

$$G\frac{Mm}{r^2} = \frac{Mm}{M+m} \cdot \frac{v^2}{r}$$

$$\Rightarrow \frac{G}{r^2} = \frac{v^2}{r(M+m)}$$

Combining Kepler’s 3rd Law and Newton’s Law of Gravitation

Simplify for v^2:

$$v^2 = \frac{G(M+m)}{r} \quad \text{— (1)}$$

Next, by representing velocity in terms of period, we get the reformulation of Kepler’s 3rd law.

$$v = \frac{2\pi r}{T}$$

$$v^2 = \left(\frac{2\pi r}{T}\right)^2 = \frac{4\pi^2 r^2}{T^2} \quad \text{— (2)}$$

Equating (1) and (2),

$$\frac{G(M+m)}{r} = \frac{4\pi^2 r^2}{T^2}$$

$$\Rightarrow T^2 = \frac{4\pi^2 r^3}{G(M+m)}$$

Newton's Reformulation of Kepler's Third Law

Since Kepler's 3rd Law applies to elliptical orbits, radius is constantly changing so we cannot use an instantaneous r. Hence, we use the semimajor axis, a. (This is the average semimajor axis of the orbit of both stars)

Newton's reformulation of Kepler's 3rd Law: $T^2 = \frac{4\pi^2 a^3}{G(M+m)}$

Where,
T is the orbital period (s)
G is the gravitational constant (6.6710-11 N m2 kg-2)
M and **m** are the masses of each body (kg)
a is the length of the semi-major axis (m)

Simplified version for solar systems: $T^2 = \frac{a^3}{M+m}$

Where,
T is the orbital period (years)
M and **m** are the masses of each body (solar masses M☉)
a is the length of the semi-major axis (astronomical units AU)

06

Doppler Effect

Equation of Doppler effect

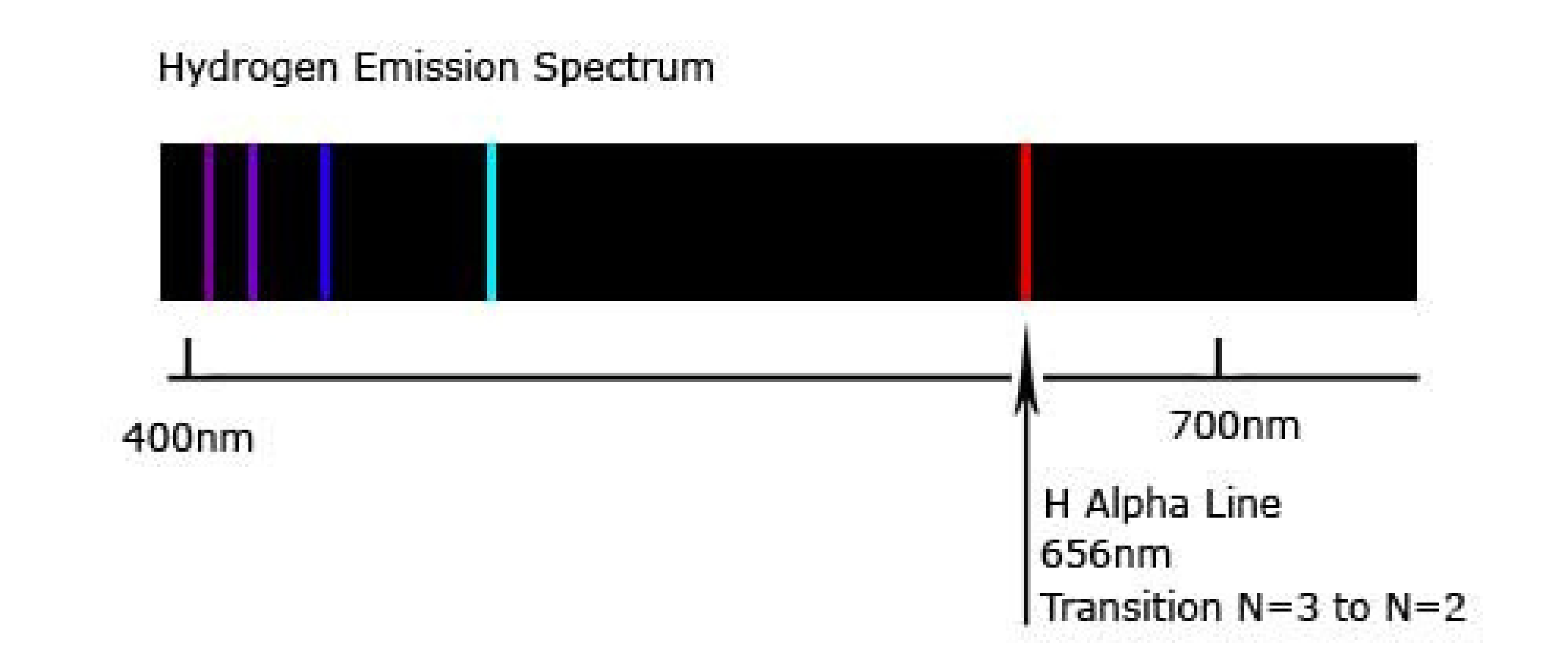
The most typical Doppler effect is pertaining to sound. However, since we are investigating Binary Stars, we have to use the version for light. The equation is as follows.

$$v_r = c \cdot \frac{\Delta\lambda}{\lambda_0}$$

If the radial velocity is -, the body is approaching the observer.
If the radial velocity is +, the body is receding from the observer

Hydrogen Alpha lines

- Hydrogen alpha lines is the deep red waves emitted or absorbed by a star observed when energy is released from an electron in a hydrogen atom.
- The spectral line occurs when an electron of a hydrogen atom drops from energy level n=3 to energy level n=2. (The greater the n number, the larger the from the nucleus of the atom.)



Wavelength 656.28nm.

07

Conservation of Momentum

Conservation of Momentum

$$M_1 v_1 = M_2 v_2 \longrightarrow \frac{M_1}{M_2} = \frac{v_2}{v_1}$$

- The Conservation of Momentum states that the total Momentum of an isolated system remains constant over time
- In a system like the Binary stars system, it falls under a special case
- When two objects of M1 and M2 are involved in an interaction where one object is initially at rest, or when considering the momentum of two objects that are moving together as a single system after an interaction.

08

Application

Worked example

We will be applying the concepts we have learnt previously to find the stellar masses of Sirius A and Sirius B in the Sirius Binary Star system.

Worked example

1. Find the sum of the masses of the two stars by finding the semimajor axis of the orbit and the orbital period.

- The semimajor axis of the Sirius binary star system is approx 20 AU
- Its period is 50 years
- The total mass of the 2 stars is 3.2 M☉
- Use Newton's Reformulation of Kepler's 3rd Law

$$T^2 = \frac{a^3}{M+m}$$

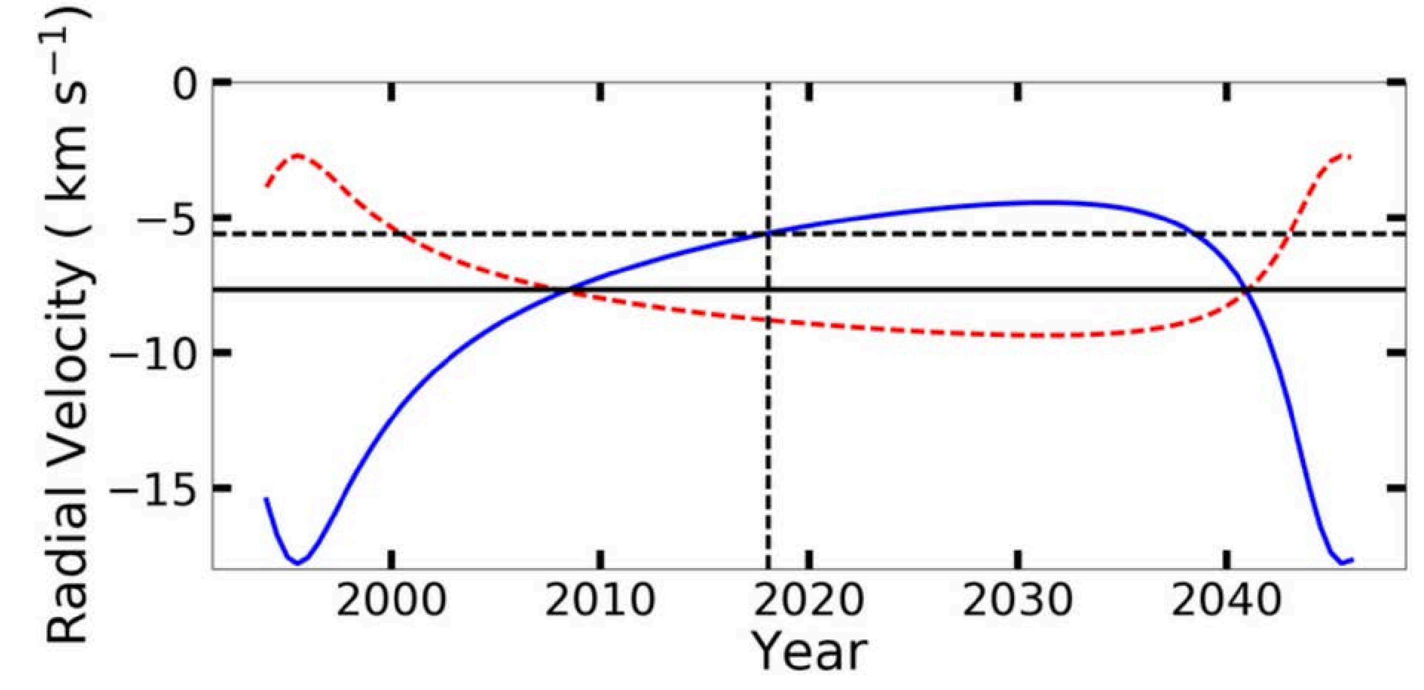
$$(20)^3 = (M+m)(50)^2$$

$$8000 = (M+m)(2500)$$

$$M+m = \frac{8000}{2500} = 3.2$$

Worked example

2. Find the Doppler shift of Sirius A and B

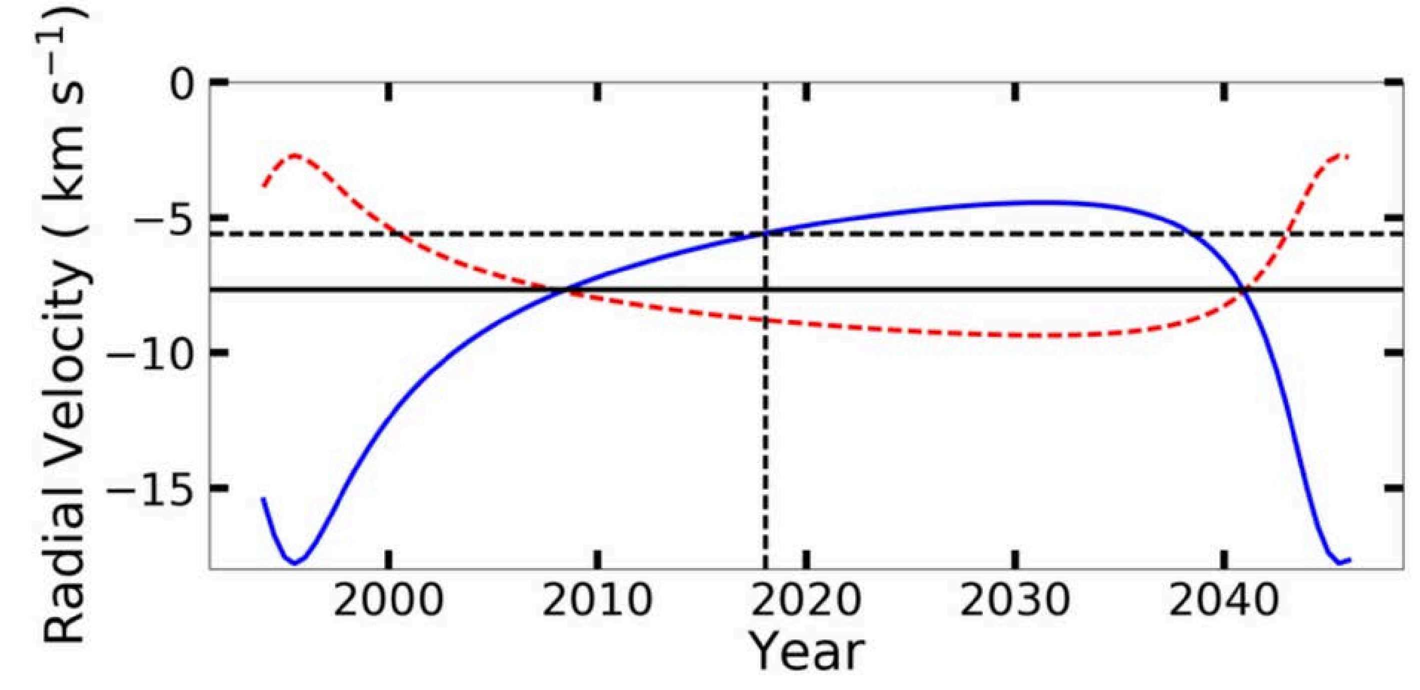


Velocity of Sirius A and B at the time of the 2018 observations. The dashed red curve (Sirius A) and blue curve (Sirius B) show the velocity of each star relative to the observer. The dashed black line marks the Sirius B velocity during the 2018 observations. The solid horizontal line is the velocity of the binary centre of mass (γ).

This is a data off a 2018 observation, recording the latest Doppler shifts of Sirius A and B

Worked example

3. Find the specific velocities of A and B



Velocity of Sirius A and B at the time of the 2018 observations. The dashed red curve (Sirius A) and blue curve (Sirius B) show the velocity of each star relative to the observer. The dashed black line marks the Sirius B velocity during the 2018 observations. The solid horizontal line is the velocity of the binary centre of mass (γ).

This is a data off a 2018 observation, recording the latest Doppler shifts of Sirius A and B

- **The observed λ of Sirius A is 2.4217 pm**
- **The observed λ of Sirius B is 4.5743 pm**

Worked example

3. Find the specific velocities of A and B

- Velocity of barycenter: -7.687
- Observed radial velocity of A: -8.794
- Observed radial velocity of B: -5.596
- VA = -1.107
- VB = +2.091

All data in km/s

Worked example

4. Solve for individual mass simultaneously

Eqn 1: $M_A + M_B = 3.2$

Eqn 2: $M_A \times (1.107) = M_B \times (2.091)$

Note: Absolute values of the velocities are used as mass cannot be negative

After computation, we get the mass of Sirius A as 2.09 M☉ and Sirius B as 1.11 M☉

Citations

BYJU'S. (n.d.). Universal law of gravitation. BYJU'S. https://byjus.com/physics/universal-law-of-gravitation/
Core. (n.d.). Binary stars. CORE. https://core.ac.uk/reader/153562167
European Space Agency. (n.d.). Orbits and Kepler's laws. NASA Science. https://science.nasa.gov/resource/orbits-and-keplers-laws/
Joyce, S. R. G., Barstow, M. A., Holberg, J. B., Bond, H. E., Casewell, S. L., & Burleigh, M. R. (2018). The gravitational redshift of Sirius B. Monthly Notices of the Royal Astronomical Society, 481(2), 2361–2370. https://academic.oup.com/mnras/article/481/2/2361/5090416?login=false
JPL. (2020, December 17). The history of Johannes Kepler. NASA Jet Propulsion Laboratory. https://www.jpl.nasa.gov/news/the-history-of-johannes-kepler/
Kloster Maulbronn. (n.d.). Johannes Kepler. Kloster Maulbronn. https://www.kloster-maulbronn.de/en/interesting-amusing/figures/johannes-kepler
Lumen Learning. (n.d.). Measuring stellar masses. Lumen Learning. https://courses.lumenlearning.com/towson-astronomy/chapter/measuring-stellar-masses/#:~:text=(The%20symbol%20~%20means%20the%20two,approximately%2081%20times%20less%20luminous.&text=The%20mass%2Dluminosity%20formula%20can,if%20the%20luminosity%20is%20known.&text=In%20the%20previous%20section%2C%20we,mass%20of%20each%20individual%20star.&text=The%20mass%20of%20the%20companion,stars%20that%20%E2%80%9Cdisobey%E2%80%9D%20it.
Math Monks. (n.d.). Ellipse. Math Monks. https://mathmonks.com/ellipse
NASA. (n.d.). Basics of space flight: Planetary orbits. NASA Science. https://science.nasa.gov/learn/basics-of-space-flight/chapter5-1/
NASA Hubble Mission Team. (2005, December 13). The Dog Star, Sirius, and its tiny companion. NASA. https://science.nasa.gov/missions/hubble/astronomers-use-hubble-to-weigh-dog-stars-companion/
Pressbooks. (n.d.). Measuring stellar masses. Pressbooks. https://pressbooks.online.ucf.edu/ast2002tjb/chapter/18-2-measuring-stellar-masses/#:~:text=Therefore%2C%20it%20moves%20more%20slowly,those%20in%20our%20immediate%20neighborhood.&text=Therefore%2C%20the%20sum%20of%20masses,to%20our%20line%20of%20sight.
Pressbooks. (n.d.). Newton's universal law of gravitation. Pressbooks. https://pressbooks.online.ucf.edu/astronomybc/chapter/3-3-newtons-universal-law-of-gravitation/
Space.com. (n.d.). Binary stars: Facts, definition & discovery. Space.com. https://www.space.com/22509-binary-stars.html
Tychos.info. (n.d.). Image of Sirius A & B. Tychos.info. https://www.google.com/url?sa=i&url=https%3A%2F%2Fwww.tychos.info%2Fchapter-4%2F&psig=AOvVaw2OnL061tbSfcWERm1EnD3D&ust=1756428084485000&source=images&cd=vfe&opi=89978449&ved=0CBgQjhxqFwoTCPi3--yirI8DFQAAAAAdAAAAABAE
Yale University. (n.d.). Measuring black holes using binary stars. Yale Astronomy. http://www.astro.yale.edu/buxton/bhcourse/tutorials/tut4/summary_4.html
YouTube. (n.d.). Kepler's third law explained. YouTube. https://www.youtube.com/watch?v=CCsbSq9wlyI

Thank you!