2017 Sec 3 Advanced Physics Chapter 1202

Dynamics: Linear Momentum

Name: ____________________ ( ) Class: 3 / ___

1202 Dynamics

- Linear momentum
- Conservation of linear momentum

Specific Instructional Objectives

(a) define and use linear momentum as the product of mass and velocity

(b) define and use impulse as the product of force and time of impact

(c) relate resultant force to the rate of change of momentum

(d) state the principle of conservation of momentum

(e) apply the principle of conservation of momentum to solve simple problems including inelastic and (perfectly) elastic interactions between two bodies in one dimension

***Note**: The following would **NOT** be assessed:*

- *Solving simultaneous equations involving conservation of energy for elastic equations*
- *The relative speed of approach and relative speed of separation*

References:

- See **momentum** at http://www.physicsclassroom.com/class/momentum
- See phET collision lab simulation at https://phet.colorado.edu/sims/collision-lab/collision-lab_en.html
- See simulation at http://www.walter-fendt.de/html5/phen/collision_en.htm
- See http://hyperphysics.phy-astr.gsu.edu/hbase/index.html
    - select **Mechanics** -> select **Conservation of momentum**: focus on **Collisions** & **Momentum**

1 Linear momentum *p*

- Linear momentum is defined as the <u>product</u> of the mass and the velocity of a body.
- Momentum, $p = mv$ where m = mass of body, v = velocity of body
- It is a vector. It has units of kg m s^{-1}.

Example 1

Calculate the magnitude of the momentum of

(a) an alpha particle of mass 6.6×10^{-27} kg travelling with a speed of 2.0×10^{7} m s^{-1}.

$p = mv = (6.6\times10^{-27})(2.0\times10^{7}) = 1.32\times10^{-19} \approx 1.3\times10^{-19}$ kgms^{-1}

(b) an oil tanker of mass 50 000 tonnes travelling with a speed of 50 m s^{-1}.

[1 tonne = 1000 kg] $p = mv = (50000\times10^{3})(50)$

$= 2.5\times10^{9}$ kgms^{-1}

(c) a Formula One car of mass 1000 kg travelling at a speed of 300 m s^{-1}.

$p = mv = (1000)(300) = 3.0\times10^{5}$ kgms^{-1} << (b)

[(a) 1.3×10^{-19} kg m s^{-1}]

Example 2

m_1 m_2

A light body and a heavy body have the same linear momentum. Which has the greater kinetic energy?

$$k.e. = \frac{1}{2}mv^2 = \frac{(mv)^2}{2m} = \frac{p^2}{2m}$$

$k.e. \propto \frac{1}{m}$ for same p.

The light body has greater k.e.

OR

$(k.e.)_1 = \frac{p^2}{2m_1}$

$(k.e.)_2 = \frac{p^2}{2m_2}$

$\frac{(k.e.)_1}{(k.e.)_2} = \frac{p^2}{2m_1} \div \frac{p^2}{2m_2}$

$= \frac{m_2}{m_1} > 1$

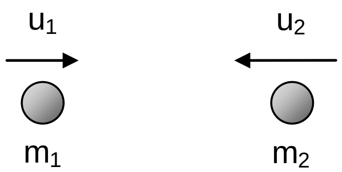
[the smaller mass has greater K.E.]

$\therefore \underbrace{(k.e.)_1}_{\text{light body has greater k.e.}} > (k.e.)_2$

2 Principle of conservation of momentum

- states that total momentum of a system remains constant if the resultant force (or net force) on the system is zero.
- Consider two particles A and B of mass m_1 and m_2 respectively, making a direct, head-on collision.
- Choose a **sign convention**: to the right is positive.

Before:



total momentum = $m_1 u_1 - m_2 u_2$

After:

total momentum = $- m_1 v_1 + m_2 v_2$

By the principle of conservation of momentum,

total momentum before collision = total momentum after collision

$$\boxed{m_1 u_1 - m_2 u_2 = - m_1 v_1 + m_2 v_2}$$

$$m_1 (u_1 + v_1) = m_2 (u_2 + v_2)$$

3 Elastic & inelastic interactions

- If a collision is elastic, the **total kinetic energy** before the collision is **equal** to the total kinetic energy after the collision. This means there is **conservation of kinetic energy**.
- If a collision is inelastic, the total kinetic energy before the collision is **not equal** to the total kinetic energy after the collision. In such a collision, kinetic energy is not conserved; it is transformed into heat, sound and/or other forms of energy.
- Although kinetic energy may or may not be conserved in a collision,
  - o linear momentum is always conserved, and
  - o total energy is always conserved.

Example 3

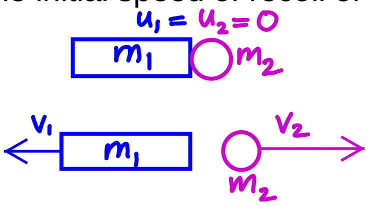
A cannon of mass 1.5 tonnes fires a cannon-ball of mass 5.0 kg. The speed with which the ball leaves the cannon is 70 m s^{-1} relative to the Earth.

m_1 m_2

Determine the initial speed of recoil of the cannon.

Before:

After:



[By principle of conservation of momentum]

$$0 + 0 = m_1(-v_1) + m_2 v_2$$

$$0 = (1.5\times10^3)(-v_1) + (5)(70)$$

$$v_1 = \frac{5(70)}{1500} = 0.2333 \simeq 0.23 \text{ ms}^{-1}$$

[0.23 m s^{-1}]

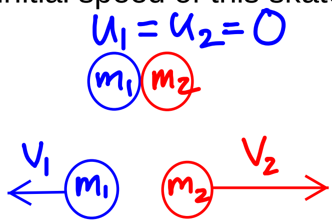
Example 4

An ice-skater of mass 80 kg, initially at rest, pushes his partner, of mass 65 kg, away from him so that she moves with an initial speed of 1.5 m s^{-1}.

Determine the initial speed of this skater after this manoeuvre.

Before:

After:



$$0 = m_1(-v_1) + m_2 v_2$$

$$0 = 80(-v_1) + 65(1.5)$$

$$v_1 = \frac{65(1.5)}{80} = 1.219$$

$$\simeq 1.2 \text{ ms}^{-1}$$

[1.2 m s^{-1}]

4 Impulse

- Impulse is the <u>product</u> of the **average force** on a body and the **time of impact** (*the time for which the force was acting*).
- Impulse = $F\,\Delta t$ where F = average force, Δt = the time of impact

5 Resultant force & rate of change of momentum

- From Newton's 2nd law of motion,
  resultant force or net force: $F_{net} = ma$

  ➔ $F = m\,\dfrac{v - u}{\Delta t}$ ➔ $F = \dfrac{mv - mu}{\Delta t}$

  Since (mv – mu) = change of momentum
  Hence, resultant force is equal to the rate of change of momentum.

$$\boldsymbol{F_{net} = \frac{mv - mu}{\Delta t}}$$

<u>Newton's 2nd law of motion</u> may also be stated as:

- **The resultant force on a body is equal to the <u>rate of change of its momentum</u>.**

- Also, $F\,\Delta t = mv - mu$ and impulse = $F\,\Delta t$
- Hence, **impulse = change in momentum = Δp**

$$\boldsymbol{\textit{impulse} = F\,\Delta t = mv - mu}$$

This formula would be given in EOY Exams.

Example 5

Show that if the net force on a system is zero, the total momentum of the system is constant.

$F_{net} = \frac{mv-mu}{\Delta t}$

If $F_{net}=0$, $mv-mu=0$, → Change in momentum of a system is zero. Hence total momentum of a system is constant.

Example 6

A safety feature of modern cars is the air-bag, which in the event of a collision, inflates and is intended to decrease the risk of serious injury. Use the concept of impulse to explain why an air bag might have this effect.

During a collision, the momentum of a passenger decreases from p=mu to zero. Impulse = change in momentum of the passenger

$F\Delta t = \Delta p$

The air bag increases the time of impact Δt experienced by the passenger. For the same impulse Δp, the force of impact F becomes smaller, reducing the risk of injury.

Example 7

Some tennis players can serve the ball at a speed of 55 m s^{-1}. The tennis ball has a mass of 60 g. In an experiment using high speed camera and video tracker, it is determined that the ball is in contact with the racket for 25 ms during the serve.

m (in kg) v Δt (in s)

Calculate the average force exerted by the racket on the ball.

$F\Delta t = mv - mu$

$F = \frac{mv-mu}{\Delta t} = \frac{\left(\frac{60}{1000}\right)(55) - 0}{25\times10^{-3}}$

$= 132\ N$ or $130\ N$ (2 s.f.)

[130 N]

Example 8

An insect of mass 4.5 mg, flying with a speed of 0.12 m s^{-1}, encounters a spider's web, which brings it to rest in 2.0 ms. Calculate the force exerted by the insect on the web.

m (in kg) Δt (in s)

$F = \frac{|mv-mu|}{\Delta t}$ magnitude only

$= \frac{|0-(4.5\times10^{-3}\times10^{-3})(0.12)|}{2.0\times10^{-3}}$

$= 2.7\times10^{-4}\ N$

4.5 mg $= 4.5\times10^{-3}$ g $= 4.5\times10^{-3}\times10^{-3}$ kg

~~*[0.27 N]*~~

Example 9

When a space rocket is taking off, the propellant gases are expelled from the rocket at a rate of 900 kg s^{-1} and at a speed of 40 km s^{-1}. Calculate the thrust exerted on the rocket.

v (in m s^{-1})

Thrust exerted on rocket = force exerted on expelled gases (by Newton's 3rd law of motion)

$F = \frac{mv-mu}{\Delta t} = \left(\frac{m}{\Delta t}\right)(v-0)$

$= (900)(40\times10^3) = 3.6\times10^7\ N$

[3.6 x 10^7 N]