2017 Sec 3 Advanced Physics Chapter 1204

Gravitational Field

Name: ____________________ ( ) **Class: 3 /** ___

1204 Gravitational field

- Gravitational field
- Gravitational force between point masses
- Gravitational field of a point mass
- Gravitational field near to the surface of the Earth
- Circular orbits

Specific Instructional Objectives

(a) show an understanding of the concept of a gravitational field as an example of field of force

(b) define the gravitational field strength at a point as the gravitational force exerted per unit mass placed at that point

(c) recall and use Newton's law of gravitation in the form $F = \frac{Gm_1m_2}{r^2}$

(d) recall and apply the equation $g = \frac{GM}{r^2}$ for the gravitational field strength of a point mass M, to new situations or to solve related problems

(e) show an understanding that near the surface of the Earth g is approximately constant and equal to the acceleration of free fall

(f) analyse circular orbits in inverse square law fields by relating the gravitational force to the centripetal acceleration it causes

Read up:

- http://sjpo.pbworks.com/w/page/66863019/gravitation_discuss

1 Gravitational field

- How does the force 'travel' from one body to another? How does a mass 'know' that another mass is attracting it? To answers these questions, physicists introduce the idea of a **field**.
- We say that a mass creates a **gravitational field** in a region of space around it. Other masses respond to this field by having a gravitational force act on them. This field is a property of the mass.

2 Newton's law of gravitation

This law states that the **gravitational force** between two point masses

- is **directly proportional** to the product of their masses and
- **inversely proportional** to their distance apart.

$$F = \frac{Gm_1m_2}{r^2}$$

This formula would be given in MYE.

where F: gravitation force
m_1 and m_2: mass of two masses
r: distance between masses
G: universal gravitational constant
$G = 6.7 \times 10^{-11}$ N m^2 kg^{-2} (Given if needed)

- This law is an inverse square law.
- The gravitational force is always attractive.

Example 1

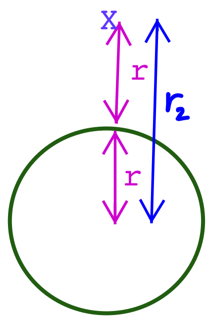
→ m　　　　of mass M

The weight of a body is 20 N on a planet's surface.
Calculate the weight of the body at a height from the surface equal to the planet's radius.

Use Equation　$W = F = \frac{GMm}{r^2}$

->Proportional　$W \propto \frac{1}{r^2}$　$\boxed{\text{only } r \text{ changes}}$

->Ratio method　$\frac{W_2}{W_1} = \left(\frac{r_1}{r_2}\right)^2 \Rightarrow W_2 = \left(\frac{r}{2r}\right)^2 \times 20$



[5.0 N]

3 Gravitational field strength *g*

- Gravitational field strength at a point is the **gravitational force** exerted **per unit mass** placed at that point. This is a vector.
- $g = \frac{F}{m}$
- Consider the gravitational field created by a spherical mass M.
- A point mass m placed a distance r from the centre of M will experience a force

$$F = \frac{GMm}{r^2}$$

- The gravitational field strength at a point a distance r from M is

$$g = \frac{GM}{r^2}$$

Example 2

The mass of Jupiter is 1.9×10^{27} kg (M) and its radius is 7.1×10^{7} m (r).
Calculate the gravitational field strength at the surface of Jupiter.

$$g = \frac{GM}{r^2}$$

$$= \frac{(6.7\times10^{-11})(1.9\times10^{27})}{(7.1\times10^{7})^2}$$

(2 s.f.)

Check: the values are in required SI units!

[25 N kg^{-1}]

Example 3

The radius of the Earth is 6.4×10^{6} m (r) and the gravitational field strength at its surface is 9.8 N kg^{-1} (g).

(a) Assuming that the field is radial, calculate the mass of the Earth. M?

$$g = \frac{GM}{r^2}$$

$$M = \frac{gr^2}{G} = \frac{9.8\times(6.4\times10^{6})^2}{6.7\times10^{-11}} = 5.991\times10^{24}$$

$$\simeq 6.0\times10^{24}\ \text{kg}$$

(b) The radius of the Moon's orbit about the Earth is 3.8×10^{8} m (r). Calculate the strength of the Earth's gravitational field at this distance. g?

$$g = \frac{GM}{r^2}$$

$$= \frac{(6.7\times10^{-11})(6.0\times10^{24})}{(3.8\times10^{8})^2} = 2.784\times10^{-3}$$

$$\simeq 2.8\times10^{-3}\ \text{N kg}^{-1}$$

(c) The mass of the Moon is 7.4×10^{22} kg (m). Calculate the gravitational attraction between the Earth and the Moon. F?

$$F = \frac{GMm}{r^2}$$

$$= \frac{(6.7\times10^{-11})(5.991\times10^{24})(7.4\times10^{22})}{(3.8\times10^{8})^2}$$

OR

$$\boxed{F = mg = 7.4\times10^{22} \times 2.784\times10^{-3}}$$

[(a) 6.0×10^{24} kg, (b) 2.8×10^{-3} N kg^{-1}, (c) 2.1×10^{20} N]

4 Gravitational field of a point mass ~~near the surface of the Earth~~

- The gravitational field of a point mass is **radial**.
- We may consider the Earth as behaving like a point mass, so its gravitational field is also radial.
- The gravitational force acting on a body placed at a distance from a planet or star is often called its **weight**.
- Weight $= F = mg$ since gravitational field strength $g = \frac{F}{m}$

Note: Near the surface of Earth, the field is approximately uniform. The fields lines would be parallel.

5 Circular orbits

- Most planets in the Solar system have orbits which are nearly circular.
- Similarly, most artificial satellites moving round the Earth and natural satellites (moons) moving round the planets have almost circular orbits.
- We can apply the equations for **circular motion** and the concept of **centripetal force** for these orbits.

Example 4

m M r

The Earth orbits the Sun in a circular orbit of radius 1.5×10^{11} m.
Determine the mass of the Sun.

$$F_{net} = ma$$

The gravitational force provides the centripetal force for the circular orbit.

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

$$= mr\omega^2$$

$$= mr\left(\frac{2\pi}{T}\right)^2$$

$T = 365$ days
$= 365 \times 24 \times 60 \times 60$
$= 3.154 \times 10^7$ s

$$\frac{GM}{r^2} = \frac{4\pi^2 r}{T^2}$$

$$\therefore M = \frac{4\pi^2}{G} \times \frac{r^3}{T^2}$$

$$= \frac{4\pi^2}{6.7 \times 10^{-11}} \times \frac{(1.5 \times 10^{11})^3}{(3.154 \times 10^7)^2}$$

[2.0×10^{30} kg]