2017 Sec 3 Advanced Physics Chapter 1203

Motion in a Circle

Name: ____________________ ( ) **Class: 3 /** ___

1203 Motion in a circle

- Centripetal acceleration
- Centripetal force

Specific Instructional Objectives

(a) express angular displacement in radians

(b) show an understanding of and use the concept of angular speed to solve problems

(c) recall and use $v = r\,\omega$ to solve problems

(d) describe qualitatively motion in a curved path due to a perpendicular force, and understand the centripetal acceleration in the case of uniform motion in a circle

(e) recall and use centripetal acceleration $a = r\omega^2$, and $a = \dfrac{v^2}{r}$ to solve problems

(f) recall and use centripetal force $F = mr\omega^2$, and $F = mv^2/r$ to solve problems.

Read up:

- http://sjpo.pbworks.com/w/page/66863003/circular_motion_discuss

1 Angular displacement θ

- In circular motion, it is convenient to measure angles in **radians** rather than in degrees.
- Angle in radians $\boldsymbol{\theta = \dfrac{l}{r}}$ where l: length of arc and r: radius of circle
- length of arc $l = r\,\theta$

2 Angular speed *ω*

- Angular speed is the angle swept out by the radius per second.
- $\boldsymbol{\omega = \dfrac{\theta}{t}}$ units: radians / s
- Angular velocity is the angular speed in a given direction.
- In one revolution, $\theta = 2\pi$, $t = \mathrm{T}$ where T: period
- $\boxed{\omega = \dfrac{\theta}{t} = \dfrac{2\pi}{\mathrm{T}} = 2\pi f}$ where f : frequency

3 Linear speed *v*

- $v = \frac{l}{t}$ units: m s^{-1}
- From earlier equations, $v = \frac{l}{t} = \frac{r\,\theta}{t} = r\,\omega$
- Hence

$$\boxed{v = r\,\omega}$$

This formula would be given in MYE.

Example 1

Calculate the angular speed (i) in degrees per second, (ii) in radians per second of

(a) a fan blade rotating at 2.5 r.p.m. (*revolutions per minute*)

→ "rounds"/min

1 "round" covers 360° or 2π rad

(i) $\omega = \frac{\theta}{t} = \frac{2.5 \times 360°}{60\,s}$

(ii) $\omega = \frac{2.5 \times 2\pi}{60\,s}$

(b) the minute hand of a clock.

→ completes 1 revolution in 1 hour.

(i) $\omega = \frac{\theta}{t} = \frac{360°}{60 \times 60\,s}$

(ii) $\omega = \frac{2\pi}{60 \times 60\,s}$

[(a)(i) 15° s^{-1}, (ii) 0.26 rad s^{-1}, (b)(i) 0.10° s^{-1}, (ii) 1.7 x 10^{-3} rad s^{-1}]

3 Centripetal acceleration *a* & centripetal force *F*

- Consider a particle going around a circle of radius ***r*** with a constant speed ***v***.
- Although the speed is constant, the direction of the velocity is **always** changing, so there is **acceleration**.
- This acceleration has a magnitude $a = \frac{v^2}{r}$ and is directed towards the centre of the circle. This means that the **net force** on the particle must be directed towards the centre of the circle.

$$\boxed{a = \frac{v^2}{r}}$$

- *This formula would be given in MYE.*
- We call this net force **centripetal force** and this provides the **centripetal acceleration** of the particle.

Example 2
Calculate the acceleration of a car which is moving in a circle of radius 90 m and has a constant speed of
$r =$
$= v$ (linear speed)

(a) 10 m s^{-1}

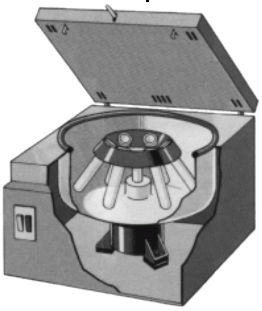
$$a = \frac{v^2}{r} = \frac{10^2}{90} \text{ ms}^{-2}$$

(b) 20 m s^{-1}

$$a = \frac{v^2}{r} = \frac{(20)^2}{90} \text{ ms}^{-2}$$

[(a) 1.1 m s^{-2}, (b) 4.4 m s^{-2}]

Example 3
A centrifuge is commonly found in a laboratory in which solid or liquid particles of different densities are separated by rotating them in a tube in a horizontal circle.



From http://www.ktf-split.hr/glossary/en_o.php?def=centrifuge

g = acceleration due to gravity
$= 10 \text{ ms}^{-2}$

A centrifuge is required to give an acceleration of 1000g to a particle at a distance of 8.5 cm from the axis of rotation. Find the necessary angular speed of the centrifuge. $= \omega$?

$r = \frac{8.5}{100}$ m

$$a = \frac{v^2}{r} = \frac{(r\omega)^2}{r}$$
$$= r\omega^2$$

$$\therefore \omega = \sqrt{\frac{a}{r}} = \sqrt{\frac{1000g}{r}} = \sqrt{\frac{1000 \times 10}{0.085}} \text{ rad s}^{-1}$$

[340 rad/s]

Examples of circular motion & centripetal force

- A car making a circular bend or turn – the centripetal force is provided by friction between the road and the tyres of the car.
- The Earth revolving around the Sun – the centripetal force is provided by the gravitational attraction force exerted on the Earth by the Sun.

Example 4

A geosynchronous satellite makes a complete circular orbit around Earth once every 24 hours. The radius of the orbit is 4.2×10^7 m. $= r$

$T = 24 \times 60 \times 60$ s

(a) What is its angular speed? $\omega = \frac{\theta}{t} = \frac{2\pi}{T} = \frac{2\pi}{24 \times 60 \times 60}$ rad s^{-1}

(b) What is its speed? $V = r\omega = 4.2 \times 10^7 \times 7.272 \times 10^{-5}$

$= 3054 \text{ ms}^{-1}$

keep more s.f. for intermediate calculations

(c) What is its centripetal acceleration? $\simeq 3100 \text{ ms}^{-1}$ (2 s.f.)

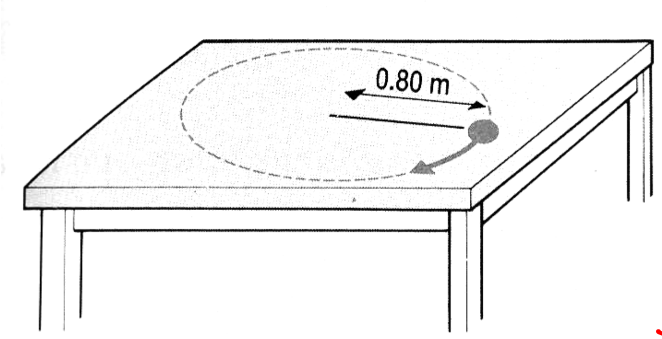
$a = \frac{v^2}{r}$ or $r\omega^2$

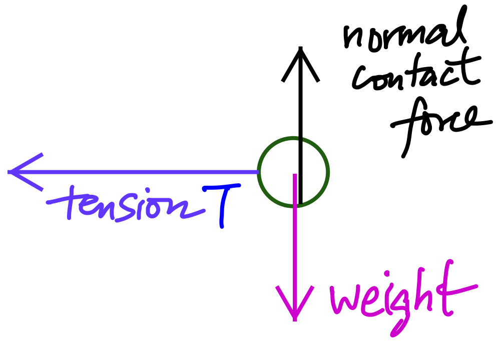
$= \frac{(3054)^2}{4.2 \times 10^7} \text{ ms}^{-2}$

[(a) 7.3×10^{-5} rad s^{-1}, (b) 3.1 km s^{-1}, (c) 0.22 m s^{-2}]

Example 5:

The figure below shows a mass of 0.300 kg (m) rotating in a circular path of radius 0.80 m (r) on a friction-free table. It is attached by a string to a peg at the centre of the circle.





(a) Draw a free body diagram for the mass.

(b) Find the force which the string exerts on the mass when the mass is moving at a constant speed of 3.45 m/s. (v)

Centripetal force

$F_{net} = ma$

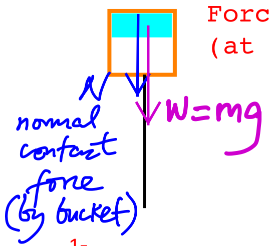
$T = \frac{mv^2}{r} = \frac{(0.300)(3.45)^2}{0.80}$

[4.46 N]

Example 6

A rope is tied to a bucket of water, and the bucket is swung in a vertical circle of radius 1.2 m. Determine the minimum speed of the bucket at the highest point of the circle if the water is to stay in the bucket throughout the motion.

Forces on water (at highest point)



$F_{net} = ma$

$N + W = \frac{mv^2}{r}$

$N = \frac{mv^2}{r} - mg$

For water to just stay in bucket, $N \geq 0$

$\frac{mv^2}{r} - mg \geq 0 \Rightarrow v \geq \sqrt{rg}$

$= \sqrt{1.2(10)}$

[~~3.4~~ m s^{-1}]
3.5