2017 Sec 3 Advanced Physics Chapter 1201

Kinematics: Non-linear Motion

Name: ______________________ (   ) Class: 3 / ___

1201 Kinematics

- Non-linear motion

Specific Instructional Objectives

(a) represent a velocity vector as two perpendicular components

(b) describe and explain motion due to a uniform velocity in one direction and a uniform acceleration in a perpendicular direction.

References:

- See **projectile motion** at http://www.physicsclassroom.com/class/vectors
- See phET projectile motion simulation at https://phet.colorado.edu/sims/html/projectile-motion/latest/projectile-motion_en.html
- See http://hyperphysics.phy-astr.gsu.edu/hbase/index.html
  - select Mechanics -> select **Newton's Laws**: focus on **Force of gravity** -> Horizontal launches & Range of Projectile

1 Projectile motion

- A projectile is an object that moves in **two dimensions** under the influence of **gravity** and nothing else.
- If air resistance is negligible, any projectile will follow the same type of path: a trajectory with the mathematical form of a parabola, of the form $y = ax^2 + bx + c$, where a is negative.
- Examples of projectile motion: balls flying through the air, long jumpers, and cars doing stunt jumps.

2 Two perpendicular components of projectile motion

- A projectile's motion can be analyzed as two perpendicular components.

	vertical component	**horizontal component**
displacement	y *(indicate sign convention)*	x
acceleration a	$a_y = 10\text{ m s}^{-2}$ acceleration due to gravity (if near Earth's surface) (acceleration under free-fall)	$a_x = 0\text{ m s}^{-2}$ no acceleration
velocity v	$v_y = u_y + a_y t$ varying velocity	$v_x = u_x$ constant velocity
approach	Apply equations of motion for constant acceleration $y = u_y t + ½ a_y t^2$ $v_y^2 = u_y^2 + 2 a_y y$	velocity = displacement / time $u_x = \frac{x}{t}$ or $x = u_x t$

3 Problem solving approach

(1) **Sketch** a diagram to show the complete path of a projectile and indicate given and unknown variables.

(2) Analyze the motion in two perpendicular directions **independently** using suitable symbols with suitable subscripts, e.g. x and y.

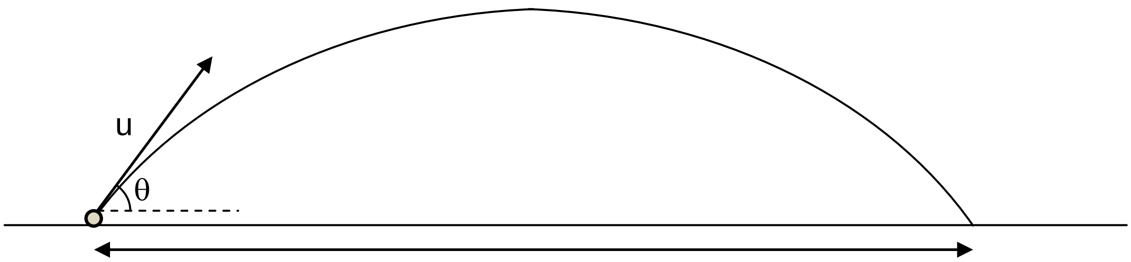
(3) Neglecting air resistance, any projectile experiences

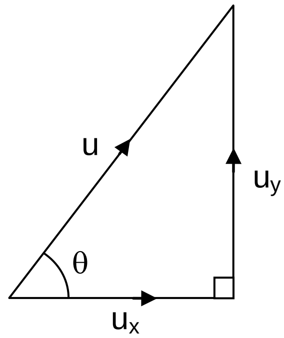
- a vertical acceleration: choose <u>sign convention</u>, apply <u>equations of motion</u> to vertical motion, &
- no horizontal acceleration (constant horizontal velocity).

(4) If the instantaneous velocity (or direction) of the projectile is needed, add the two components of the velocity (v_x and v_y) using vector addition.

4 Launch angle

- A projectile is launched with an initial speed u at an angle θ to the horizontal, as shown below.





- o the x-component of the initial velocity is $u_x = u \cos\theta$
- o the y-component of the initial velocity is $u_y = u \sin\theta$
- o $u^2 = u_x^2 + u_y^2$
- o $\tan\theta = \dfrac{u_x}{u_y}$ *(Ensure calculator in degree mode)*

5 Range

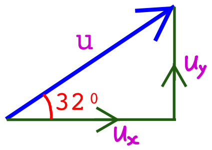
- Range is the horizontal distance travelled by a projectile.
- range x = horizontal velocity × time

  $x = u_x t$
- It can be shown that the **maximum range** is obtained with a launch angle of 45°.

6 Effect of air resistance on projectile motion

- reduce the maximum height
- reduce the maximum range
- make the angle of descent steeper
- distort the shape of the path away from a parabola.

Example 1
A ball is thrown at an angle of 32° from the ground with a speed of 25 m s^{-1}. Calculate the magnitude of the horizontal and vertical components of its initial velocity.



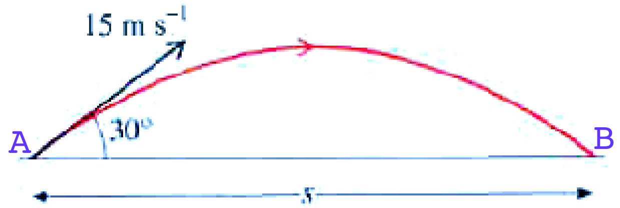
Vertical component: $u_y = u\sin\theta$
$= 25\sin 32°$
$= 13.25 \simeq 13\ \text{ms}^{-1}$

Horizontal component: $u_x = u\cos\theta$
$= 25\cos 32°$
$= 21.20 \simeq 21\ \text{ms}^{-1}$

- *Note: Check u =*

[21 m s^{-1}, 13 m s^{-1}]

Example 2



A football is kicked on level ground at a velocity of 15 m s^{-1} at an angle of 30° to the horizontal. Determine the horizontal distance travelled by the football till its first bounce on the ground.

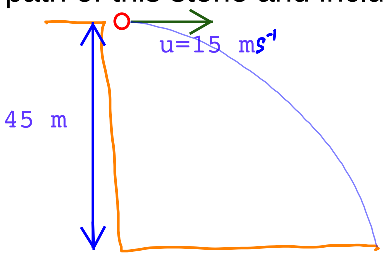
- *sign convention:* up is positive, $a = -g = -10\ \text{ms}^{-1}$
- *vertical motion:* $y = u_y t + \frac{1}{2}at^2$

From A to B, $y = 0 \Rightarrow 0 = (u\sin\theta)t - \frac{1}{2}gt^2$
$u\sin\theta - \frac{1}{2}gt \Rightarrow t = \frac{2u\sin\theta}{g} = \frac{2(15)\sin 30°}{10}$
$= 1.5\ s$

- *horizontal motion:* $x = u_x t = (u\cos\theta)t$
$= (15\cos 30°)(1.5) = 19.49$
$\simeq 19\ m$

[19 m]

Example 3
A stone is thrown from the top of a cliff, 45 m high above level ground, with an initial velocity of 15 m s^{-1} in a horizontal direction.
(a) Sketch the path of this stone and include the information provided above.



$\theta = 0°$
$u_x = u\cos\theta = 15\cos 0° = 15\ \text{ms}^{-1}$
$u_y = u\sin\theta = 15\sin 0° = 0\ \text{ms}^{-1}$

(b) Determine the time taken for the stone to reach the ground.
Sign convention: downwards is positive
Vertical motion: $y = u_y t + \frac{1}{2}a_y t^2$
$45 = 0 + \frac{1}{2}(10)t^2 \Rightarrow t^2 = 9$
$t = 3.0\ s$

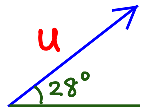
(c) Determine the distance of the stone from the base of the cliff when it reaches the ground.
Horizontal motion: $x = u_x t$
$= 15 \times 3.0 = 45\ m$

[(b)= 3.0 s, (c) 45 m]

Example 4

An athlete competing in the long jump leaves the ground at an angle of 28° and makes a jump of 7.40 m $\rightarrow x$

(a) Calculate the speed $\rightarrow u$ at which the athlete took off.



Horizontal motion: $x = u_x t$

$$7.40 = (u\cos\theta)t \Rightarrow t = \frac{7.40}{u\cos\theta} \quad (1)$$

Sign convention: upwards is positive

Vertical motion: $y = u_y t + \frac{1}{2}a_y t^2$

$$0 = (u\sin\theta)t - \frac{1}{2}(10)t^2 \Rightarrow u\sin\theta = 5t \quad (2)$$

Substitute equation (1) into (2),

$$u\sin\theta = \frac{5(7.40)}{u\cos\theta} \Rightarrow u^2 = \frac{5(7.40)}{\sin 28^\circ \cos 28^\circ} \Rightarrow u = \sqrt{89.26}$$

$$= 9.448$$

$$\simeq 9.4\ ms^{-1}$$

(b) If the athlete had been able to increase this speed $\rightarrow u$ by 5%, determine the percentage difference this would have made to the length $\rightarrow x$ of the jump.

Express x in terms of u!

From equation (2), $t = \frac{u\sin\theta}{5}$

$$x = u_x t = (u\cos\theta)\frac{u\sin\theta}{5} \Rightarrow x = \frac{u^2(\sin\theta)(\cos\theta)}{5} \Rightarrow x \propto u^2$$

$$x \propto (1.05u)^2$$

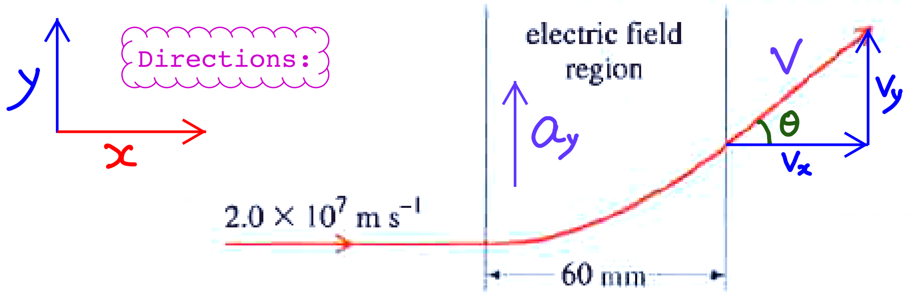
$$u \rightarrow \underbrace{1.05}_{\text{5\% increase}}u,$$

$$= 1.1025u^2$$

$$\simeq \underbrace{1.10}_{\text{10\% increase}}u^2$$

[(a) 9.4 m s^{-1}; (b) 10%]

Example 5



An electron, travelling with a velocity of 2.0 x 10^{7} m s^{-1} in a horizontal direction, enters a uniform electric field. This field gives the electron a constant acceleration of 5.0 x 10^{15} m s^{-2} in a direction perpendicular to its original velocity.

Determine the magnitude and direction of the velocity of the electron when it leaves the field.

V $\quad$ θ

Horizontal motion: $V_x = u_x = u = 2.0\times10^{7}\ ms^{-1}$

$$x = u_x t \Rightarrow t = \frac{x}{u_x} = \frac{60\times10^{-3}\ m}{2.0\times10^{7}\ ms^{-1}}$$

$$= 3.0\times10^{-9}\ s$$

Sign convention: upwards is positive!

Vertical motion:

$$V_y = u_y + a_y t = 0 + (5.0\times10^{15})(3.0\times10^{-9})$$

$$= 1.5\times10^{7}\ ms^{-1}$$

$$V = \sqrt{V_x^2 + V_y^2} = 2.5\times10^{7}\ ms^{-1}$$

$$\tan\theta = \frac{V_y}{V_x} = \frac{1.5}{2.0} \Rightarrow \theta = \tan^{-1}\left(\frac{1.5}{2.0}\right)$$

$$= 36.87$$

$$\simeq 37^\circ$$

[2.5 × 10^{7} m s^{-1}; 37° to the horizontal]