2020 Sec 4 Advanced Physics

General Wave Properties: Superposition

Name: _____ ( )   Class: 4 / ___

Superposition
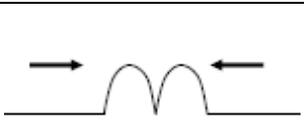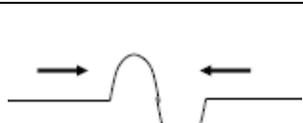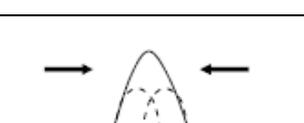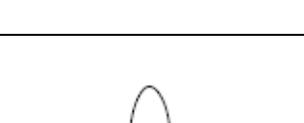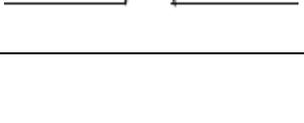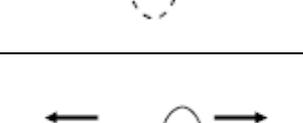- Principle of Superposition
- Stationary Waves and Progressive Waves
- Examples of Stationary Waves

Learning Outcomes

(a) explain and use the principle of superposition in simple applications
(b) show an understanding of experiments which demonstrate stationary wave using stretched strings and air columns
(c) explain the formation of a stationary wave using a graphical method, and identify nodes and antinodes
(d) determine the wavelength of sound using stationary waves

1   Principle of Superposition

- When two (or more) waves cross each other, they add up to give a resultant wave. This combination of waves follows the Principle of Superposition.

1.	Two pulses are some distance apart and approaching each other.		
2.	They are about to meet.		
3.	They are partially overlapping, the resultant is the vector sum of the two.		
4.	They are exactly overlapping each other and maximum adding/cancellation is achieved.		
5.	The pulses are receding from each other.		

The **Principle of Superposition** states that when two or more waves of the same kind meet at a point, the displacement of the resultant wave is the vector sum of the displacements of the individual waves at that point.

Example 1

Two identical wave pulses travel at a speed of 2.0 m s^{-1} towards each other on a long cord. If the pulses are 6.0 cm apart at t = 0 s, sketch the shape of the cord at t = 10, 15 and 20 ms.


6.0 cm
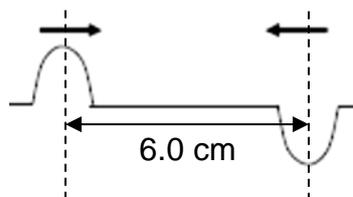
Example 2

Two waves, A and B meet at point P. The displacement-time graphs due to each individual wave at P are as shown.

Sketch the displacement-time graph of the resultant wave at P.

displacement

wave A

wave B

time
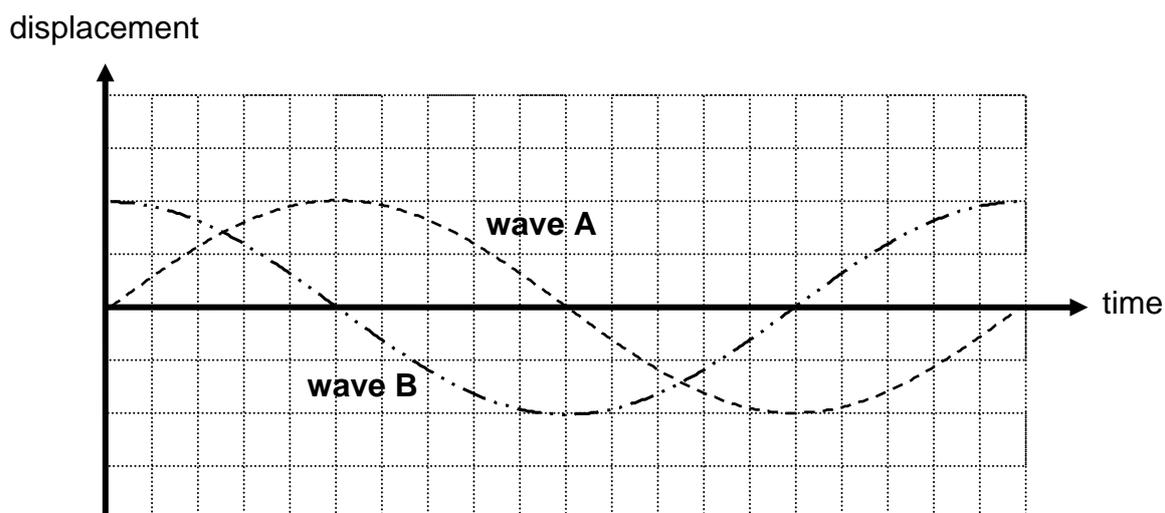
2   Stationary Waves and Progressive Waves

- **Stationary waves (or standing waves)** arise from the superposition of two progressive waves of the **same frequency and amplitude moving with the same speed in opposite directions**.

- Consider two waves (red and blue) with the same frequency and amplitude, moving with the same speed but in opposite directions:
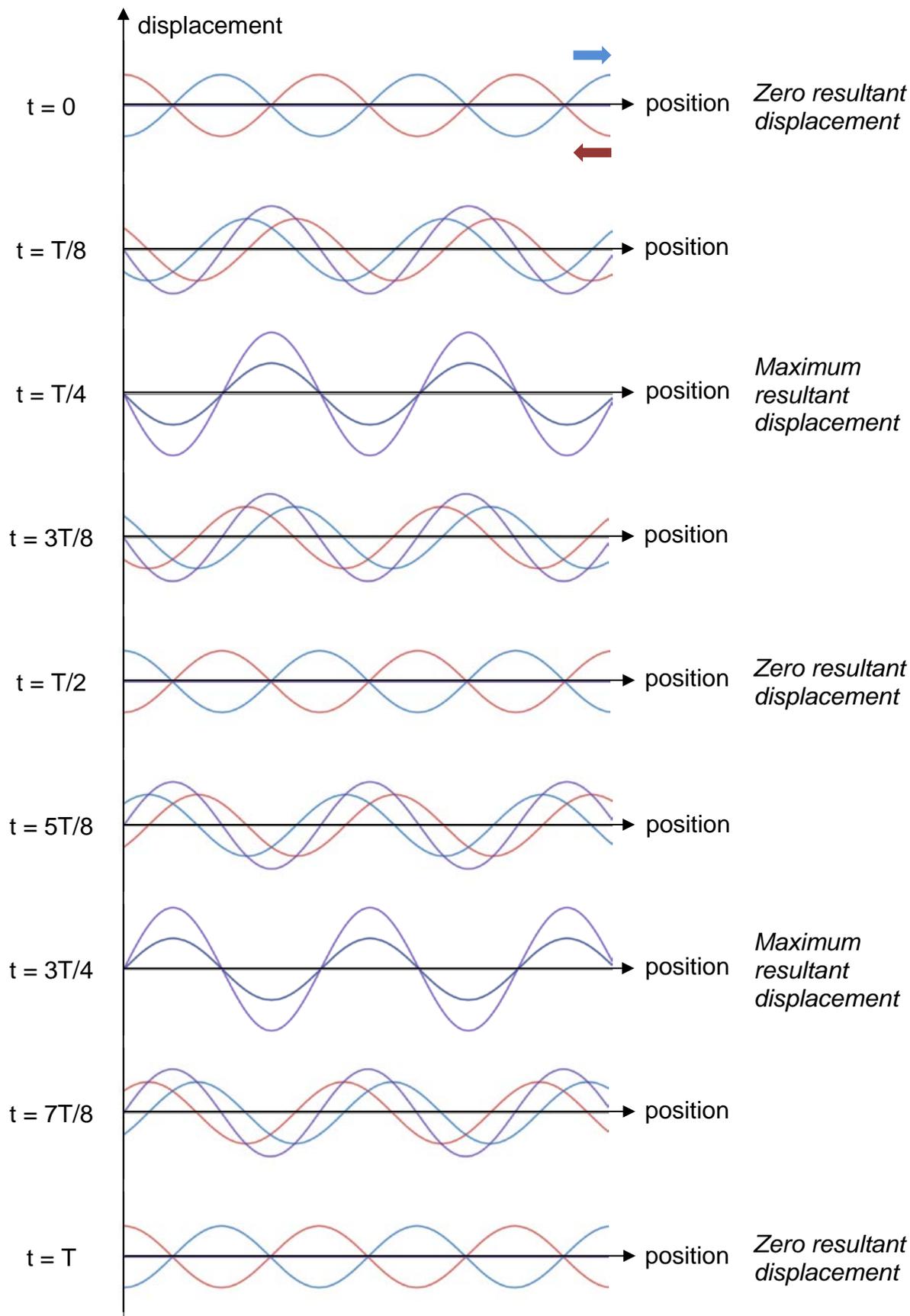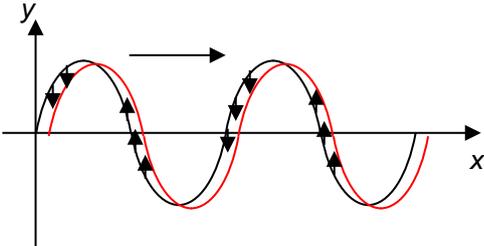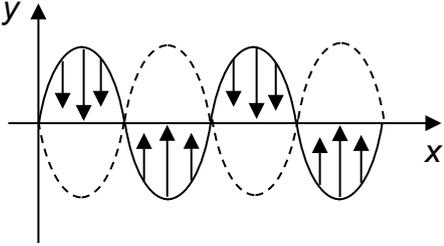
- Different positions on the stationary wave will have **different amplitudes**.

- **Nodes** are positions on the stationary wave where the **displacement is always zero**.

- **Anti-nodes** are positions on the stationary wave where the **amplitude is the largest**.

Example 3

For the graphs on page 3,

(a)   label the nodes *N* and anti-nodes *A*,

(b)   label the wavelength *λ* of the stationary wave.

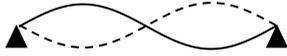- Comparing progressive and stationary waves:

	Progressive Wave	**Stationary Wave**
Wave Profile	Wave profile advances with the speed of the wave. 	Wave profile does not advance.
Energy	Energy is transported in the direction of travel of the wave.	Energy is retained within the vibratory motion of the stationary wave.
Frequency	Particles of wave vibrate with frequency of progressive wave.	Particles of wave vibrate with frequency of stationary wave.
Wavelength	Distance between any two consecutive points on the wave with the same phase.	Twice the distance between a pair of adjacent nodes or anti-nodes.
Amplitude	Same amplitude for all particles in the wave motion (if no loss of energy)	Varies from zero (at node) to a maximum (at anti-node)

- Links for further exploration:

  – Wave simulation: https://phet.colorado.edu/sims/html/wave-on-a-string/latest/wave-on-a-string_en.html

  – Superposition graphs: https://www.desmos.com/calculator/gus721cqlh

3 Examples of Stationary Waves

3.1 Stationary Waves in Stretched Strings

- If a stretched string is plucked or struck, stationary waves may be produced.

- Since the velocity of the waves along the string remains constant, a change in frequency causes the wavelength to change and **different modes of vibration** arise.

- As the ends are fixed in position, there must be a **node** at each end of the string.
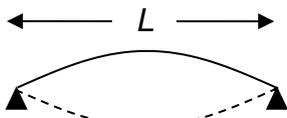
Harmonic Series	Mode of Vibration	Wavelength	Frequency
First harmonic (Fundamental mode)			
Second harmonic (First overtone)			
Third harmonic (Second overtone)			
n-th harmonic (n−1-th overtone)	All harmonics are possible		

Example 4

A wire is fixed at each end under tension. A transverse wave of speed 300 m s^{-1} is propagated along the wire and forms a standing wave pattern. In a certain mode of vibration, it is found that the nodes are 0.40 m apart.

What is the frequency of this standing wave?

*[380 Hz]*

3.2 Stationary Waves in Air Columns

- Stationary waves can also be formed in both open pipes (both ends are opened) and closed pipes (one end is closed).

- Since the velocity of sound remains constant in air, different frequencies will produce **different modes of vibration**.

- The **closed end** of the pipe is always a **node** because the layer of air molecules directly in contact with this end cannot vibrate.

- The **open end** of the pipe is always an **anti-node** since the air molecules at this end are free to vibrate.

3.2.1 Open Pipes

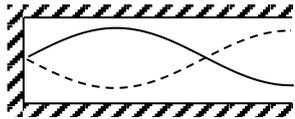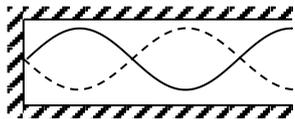| Harmonic Series	Mode of Vibration	Wavelength	Frequency
First harmonic (Fundamental mode)			
Second harmonic (First overtone)			
Third harmonic (Second overtone)			
n-th harmonic (n−1-th overtone)	All harmonics are possible		

Example 5

A speaker is placed near one end of an open pipe of length 2.5 m. When the frequency of the speaker is 260 Hz, a stationary is formed within the pipe as shown. Calculate the speed of sound.
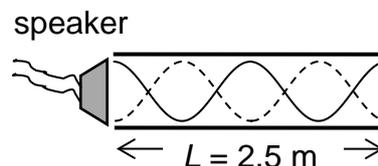

[330 m s^{-1}]

3.2.2 Closed Pipes
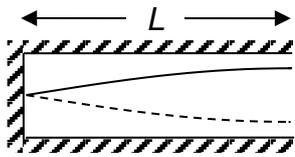
Harmonic Series	Mode of Vibration	Wavelength	Frequency
First harmonic (Fundamental mode)			
Second harmonic	-	-	-
Third harmonic (First overtone)			
Fourth harmonic	-	-	-
Fifth harmonic (Second overtone)			
n-th harmonic ($\frac{n-1}{2}$-th overtone)	Only odd-numbered harmonics are possible		

Example 6

A note of constant frequency is produced by a loudspeaker placed above an open pipe with a movable piston. The length l of the air column is slowly increased from zero. When l reaches 17.0 cm, the sound detected increases greatly in volume.
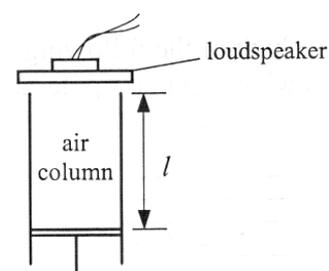
Calculate

(a)   the wavelength of the note produced by the loudspeaker, and

(b)   the next length l when the sound detected increases greatly in volume.

[68.0 cm, 51.0 cm]