



## Dynamics: Linear Momentum

Name: \_\_\_\_\_ ( ) Class: 3 / \_\_\_\_

### Linear Momentum

- Linear momentum
- Conservation of linear momentum

### Learning Outcomes

- define and use linear momentum as the product of mass and velocity
- define and use impulse as the product of force and time of impact
- relate resultant force to the rate of change of momentum
- state the principle of conservation of momentum
- apply the principle of conservation of momentum to solve simple problems including inelastic and (perfectly) elastic interactions between two bodies in one dimension

**Note:** The following would **NOT** be assessed:

- Solving simultaneous equations involving conservation of energy for elastic equations
- The relative speed of approach and relative speed of separation

### References:

- See **momentum** at <http://www.physicsclassroom.com/class/momentum>
- See phET collision lab simulation at [https://phet.colorado.edu/sims/collision-lab/collision-lab\\_en.html](https://phet.colorado.edu/sims/collision-lab/collision-lab_en.html)
- See simulation at [http://www.walter-fendt.de/html5/phen/collision\\_en.htm](http://www.walter-fendt.de/html5/phen/collision_en.htm)
- See <http://hyperphysics.phy-astr.gsu.edu/hbase/index.html>
  - select **Mechanics** -> select **Conservation of momentum**: focus on **Collisions & Momentum**

### 1 Linear momentum $p$

- Linear momentum is defined as the product of the mass and the velocity of a body.
- Momentum,  $p = mv$  where  $m$  = mass of body,  $v$  = velocity of body
- It is a vector. It has units of  $\text{kg m s}^{-1}$ .

### Example 1

Calculate the magnitude of the momentum of

- an alpha particle of mass  $6.6 \times 10^{-27} \text{ kg}$  travelling with a speed of  $2.0 \times 10^7 \text{ m s}^{-1}$ .
- an oil tanker of mass 50 000 tonnes travelling with a speed of  $50 \text{ m s}^{-1}$ .  
[1 tonne = 1000 kg]
- a Formula One car of mass 1000 kg travelling at a speed of  $300 \text{ m s}^{-1}$ .

*[(a)  $1.3 \times 10^{-19} \text{ kg m s}^{-1}$ ; (b)  $2.5 \times 10^9 \text{ kg m s}^{-1}$ ; (c)  $3.0 \times 10^5 \text{ kg m s}^{-1}$ ]*

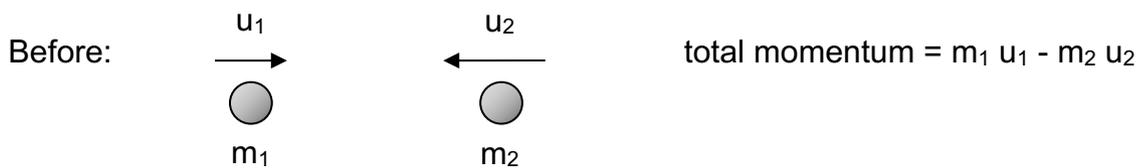
## Example 2

A light body and a heavy body have the same linear momentum. Which has the greater kinetic energy?

*[the smaller mass has greater K.E.]*

## 2 Principle of conservation of momentum

- states that total momentum of a system remains constant if the resultant force (or net force) on the system is zero.
- Consider two particles A and B of mass  $m_1$  and  $m_2$  respectively, making a direct, head-on collision.
- Choose a **sign convention**: to the right is positive.



By the principle of conservation of momentum,

total momentum before collision = total momentum after collision

$$m_1 u_1 - m_2 u_2 = - m_1 v_1 + m_2 v_2$$

$$m_1 (u_1 + v_1) = m_2 (u_2 + v_2)$$

## 3 Elastic & inelastic interactions

- If a collision is elastic, the **total kinetic energy** before the collision is **equal** to the total kinetic energy after the collision. This means there is **conservation of kinetic energy**.
- If a collision is inelastic, the total kinetic energy before the collision is **not equal** to the total kinetic energy after the collision. In such a collision, kinetic energy is not conserved; it is transformed into heat, sound and/or other forms of energy.
- Although kinetic energy may or may not be conserved in a collision,
  - linear momentum is always conserved, and
  - total energy is always conserved.

### Example 3

A cannon of mass 1.5 tonnes fires a cannon-ball of mass 5.0 kg. The speed with which the ball leaves the cannon is  $70 \text{ m s}^{-1}$  relative to the Earth.

Determine the initial speed of recoil of the cannon.

*[0.23 m s<sup>-1</sup>]*

### Example 4

An ice-skater of mass 80 kg, initially at rest, pushes his partner, of mass 65 kg, away from him so that she moves with an initial speed of  $1.5 \text{ m s}^{-1}$ .

Determine the initial speed of this skater after this manoeuvre.

*[1.2 m s<sup>-1</sup>]*

## 4 Impulse

- Impulse is the product of the **average force** on a body and the **time of impact** (*the time for which the force was acting*).
- Impulse =  $F \Delta t$                       where  $F$  = average force,  $\Delta t$  = the time of impact

## 5 Resultant force & rate of change of momentum

- From Newton's 2<sup>nd</sup> law of motion, resultant force or net force:  $F_{\text{net}} = ma$

$$\rightarrow F = m \frac{v - u}{\Delta t} \quad \rightarrow F = \frac{mv - mu}{\Delta t}$$

Since  $(mv - mu)$  = change of momentum

Hence, resultant force is equal to the rate of change of momentum.

$$F_{\text{net}} = \frac{mv - mu}{\Delta t}$$

**Newton's 2<sup>nd</sup> law of motion** may also be stated as:

- **The resultant force on a body is equal to the rate of change of its momentum.**
- Also,  $F \Delta t = mv - mu$                       and                      impulse =  $F \Delta t$
- Hence, **impulse = change in momentum =  $\Delta p$**

$$\text{impulse} = F \Delta t = mv - mu$$

*This formula would be given in EOY Exams.*

**Example 5**

Show that if the net force on a system is zero, the total momentum of the system is constant.

**Example 6**

A safety feature of modern cars is the air-bag, which in the event of a collision, inflates and is intended to decrease the risk of serious injury. Use the concept of impulse to explain why an air bag might have this effect.

**Example 7**

Some tennis players can serve the ball at a speed of  $55 \text{ m s}^{-1}$ . The tennis ball has a mass of 60 g. In an experiment using high speed camera and video tracker, it is determined that the ball is in contact with the racket for 25 ms during the serve.

Calculate the average force exerted by the racket on the ball.

*[130 N]*

**Example 8**

An insect of mass 4.5 mg, flying with a speed of  $0.12 \text{ m s}^{-1}$ , encounters a spider's web, which brings it to rest in 2.0 ms. Calculate the force exerted by the insect on the web.

*[ $2.7 \times 10^{-4} \text{ N}$ ]*

**Example 9**

When a space rocket is taking off, the propellant gases are expelled from the rocket at a rate of  $900 \text{ kg s}^{-1}$  and at a speed of  $40 \text{ km s}^{-1}$ . Calculate the thrust exerted on the rocket.

*[ $3.6 \times 10^7 \text{ N}$ ]*