2020 Sec 3 Advanced Physics

Kinematics: Non-linear Motion

Name: ____________________ (   ) Class: 3 / ___

Kinematics

- Non-linear motion

Learning Outcomes

(a) represent a velocity vector as two perpendicular components

(b) describe and explain motion due to a uniform velocity in one direction and a uniform acceleration in a perpendicular direction.

References:

- See **projectile motion** at http://www.physicsclassroom.com/class/vectors
- See phET projectile motion simulation at https://phet.colorado.edu/sims/html/projectile-motion/latest/projectile-motion_en.html
- See http://hyperphysics.phy-astr.gsu.edu/hbase/index.html
  - select Mechanics -> select **Newton's Laws**: focus on **Force of gravity** -> Horizontal launches & Range of Projectile

1 Projectile motion

- A projectile is an object that moves in **two dimensions** under the influence of **gravity** and nothing else.
- If air resistance is negligible, any projectile will follow the same type of path: a trajectory with the mathematical form of a parabola, of the form $y = ax^2 + bx + c$, where a is negative.
- Examples of projectile motion: balls flying through the air, long jumpers, and cars doing stunt jumps.

2 Two perpendicular components of projectile motion

- A projectile's motion can be analyzed as two perpendicular components.

	vertical component	**horizontal component**
displacement	y *(indicate sign convention)*	x
acceleration a	$a_y = 10\ \text{m s}^{-2}$ acceleration due to gravity (if near Earth's surface) (acceleration under free-fall)	$a_x = 0\ \text{m s}^{-2}$ no acceleration
velocity v	$v_y = u_y + a_y t$ varying velocity	$v_x = u_x$ constant velocity
approach	Apply equations of motion for constant acceleration $y = u_y t + \frac{1}{2} a_y t^2$ $v_y^2 = u_y^2 + 2 a_y y$	velocity = displacement / time $u_x = \frac{x}{t}$ or $x = u_x t$

3 Problem solving approach

(1) **Sketch** a diagram to show the complete path of a projectile and indicate given and unknown variables.

(2) Analyze the motion in two perpendicular directions **independently** using suitable symbols with suitable subscripts, e.g. x and y.

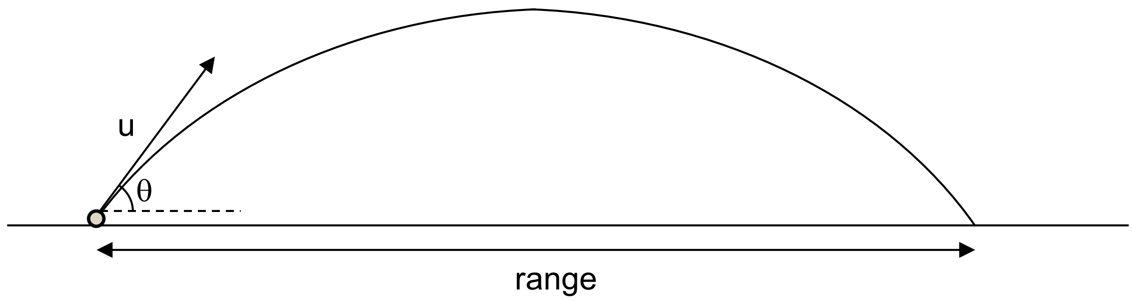
(3) Neglecting air resistance, any projectile experiences

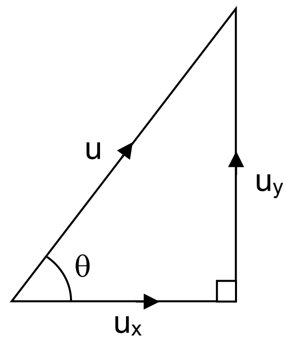
- a vertical acceleration: choose <u>sign convention</u>, apply <u>equations of motion</u> to vertical motion, &
- no horizontal acceleration (constant horizontal velocity).

(4) If the instantaneous velocity (or direction) of the projectile is needed, add the two components of the velocity (v_x and v_y) using vector addition.

4 Launch angle

- A projectile is launched with an initial speed u at an angle θ to the horizontal, as shown below.





- the x-component of the initial velocity is $u_x = u \cos \theta$
- the y-component of the initial velocity is $u_y = u \sin \theta$
- $u^2 = u_x^2 + u_y^2$
- $\tan \theta = \dfrac{u_x}{u_y}$ *(Ensure calculator in degree mode)*

5 Range

- Range is the horizontal distance travelled by a projectile.
- range x = horizontal velocity × time

  $x = u_x t$
- It can be shown that the **maximum range** is obtained with a launch angle of 45°.

6 Effect of air resistance on projectile motion

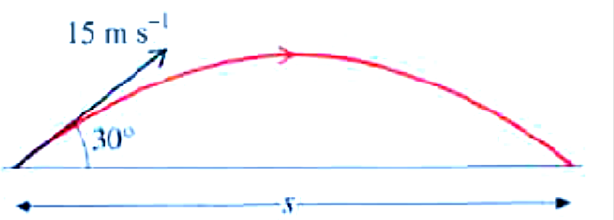
- reduce the maximum height
- reduce the maximum range
- make the angle of descent steeper
- distort the shape of the path away from a parabola.

Example 1
A ball is thrown at an angle of 32° from the ground with a speed of 25 m s^{-1}. Calculate the magnitude of the horizontal and vertical components of its initial velocity.

- *Note: Check u =*

[21 m s^{-1}, 13 m s^{-1}]

Example 2



A football is kicked on level ground at a velocity of 15 m s^{-1} at an angle of 30° to the horizontal. Determine the horizontal distance travelled by the football till its first bounce on the ground.

- *sign convention:*
- *vertical motion:*

- *horizontal motion:*

[19 m]

Example 3
A stone is thrown from the top of a cliff, 45 m high above level ground, with an initial velocity of 15 m s^{-1} in a horizontal direction.
(a) Sketch the path of this stone and include the information provided above.

(b) Determine the time taken for the stone to reach the ground.

(c) Determine the distance of the stone from the base of the cliff when it reaches the ground.

[(b)= 3.0 s, (c) 45 m]

Example 4

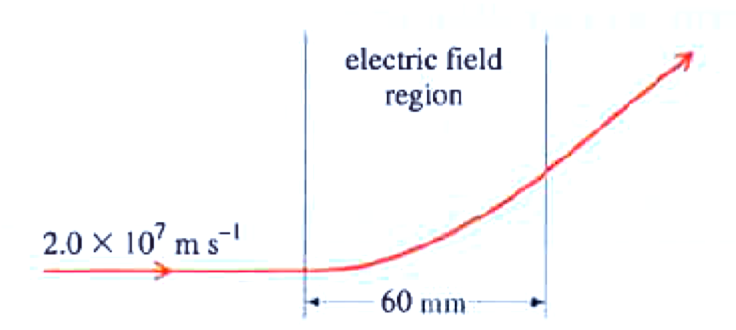
An athlete competing in the long jump leaves the ground at an angle of 28° and makes a jump of 7.40 m.

(a) Calculate the speed at which the athlete took off.

(b) If the athlete had been able to increase this speed by 5%, determine the percentage difference this would have made to the length of the jump.

[(a) 9.4 m s^{-1}; (b) 10%]

Example 5



An electron, travelling with a velocity of 2.0×10^7 m s^{-1} in a horizontal direction, enters a uniform electric field. This field gives the electron a constant acceleration of 5.0×10^{15} m s^{-2} in a direction perpendicular to its original velocity.

Determine the magnitude and direction of the velocity of the electron when it leaves the field.

[2.5×10^7 m s^{-1}; 37° to the horizontal]