Forces: Statics and Dynamics

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1 Introduction

We've discussed the most general rules for forces in the previous chapter. This chapter aims to go into the slightly more detailed portions, which are more relevant to solving the problems. After some time, you will realise the way to solve these problems is quite standard: just use the laws we've learnt in the previous chapter.

It is also good to practise a skill called physical intuition, which is to have a sense of how the system work by relating to our physical world. Does it sound commonsense enough? One example is Newton's Third Law, which I remember having a lot of problem trying to visualize when I first learnt it. Try imagining pushing your hand across the rough table. What are the forces on your hand and on the table?

2 Type of Forces

It should be quite obvious there are various types of forces, working in different ways, governed by different equations. Of course, there are also other types of forces, these are just some more common ones.

Just some trivial knowledge. There are four fundamental knowledge(at least until now, I heard some scientist might have discovered a fifth one). In order of increasing strength, they are gravitational force, weak nuclear force, electromagnetic force and strong nuclear force.

2.1 Gravitational Force

This force should be very familiar. It is basically the **attractive** force that exists between two masses. All we have to know is governed by Newton's Universal Law of Gravitation:

The gravitational force between two masses is **directly proportional** to the **product of their masses** and **inversely proportional** to the **square of the distance between them**.

The constant of proportionality is given by G, the gravitational constant.

$$F_{gravitational} = G\frac{m_1 m_2}{r^2}$$

We will come back to this equation specifically in a later chapter. But for now, I will discuss something simpler, or rather an approximation. The gravitational force close to the surface of the Earth.

The equation above shows that as you get further away from Earth, r increases, and the gravitational force gets weaker. But by how much? The radius of Earth is $6371km = 6371000m$. Let's say you are on top of Mount Everest, which is $8848m$ high, the difference in r is only 0.14%. The difference in gravitational force is negligible. It is not very far to assume the gravitational force is a constant on the surface of Earth.

$$F_{surface\,of\,Earth} = G\frac{m_{Earth} m_{object}}{r_{Earth}^2}$$

If you see, G, m_{Earth}, r_{Earth} are all constants. So, an object of mass m just experience a force $F = mg$, where g is the acceleration due to gravity on Earth, approximately $9.81\,ms^{-1}$.

$$g = G\frac{m_{Earth}}{r_{Earth}^2}$$

If you remember, in the chapter on kinematics, we assumed an object undergoing projectile motion experiences a constant downward force. This is why.

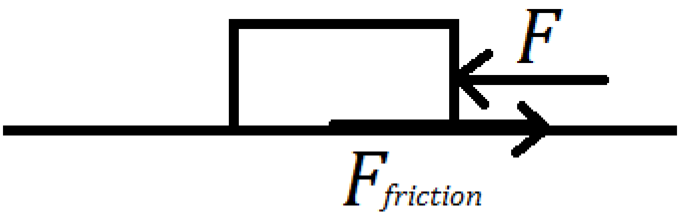
2.2 Normal Force

Let's say you are reading this set of notes on your laptop or iPad, which is placed on a table. We know that Earth is exerting a downward force on your laptop, which means your laptop is supposed to accelerate downwards at $9.81\,ms^{-1}$. (To prove this point you can try dropping your laptop on the ground.) But it is not moving downwards. Therefore we can logically conclude that the table is exerting an upward force on your table.

This type of force is called the normal force, which is a force exerted by a surface, perpendicular to the surface(The word normal usually has some relation to the idea of being perpendicular). There is not really an equation to calculate the value of this force, but we can derive its value based on the the conditions in the system. Take the example of the laptop on the table. The normal force needs to cancel out the gravitational force exactly for it to remain stationary(using Newton's 1st Law), which means the normal force is also mg.

2.3 Frictional Force

Frictional force sometimes include viscous force, air resistance, but in our case, we are specifically referring to the force parallel to a surface, opposite the direction of motion.



There are two types of friction: static and kinetic. As the name probably suggests, they are when the object is moving, and when it is not. They are more or less governed by the same equation. It is the product of a constant μ_{static} and $\mu_{kinetic}$, that depends on the type of surface, and the normal force on the surface, N.

When it remains stationary, this implies the net force on the object is zero, and that the friction force must cancel out the force exerted.

$$F_{friction} = F$$

This means when you push it with a smaller force, the frictional force is also naturally smaller. The static friction is not an exact value but really a range of value restricted by an upper bound.

$$0 \leq F_{friction} = F \leq \mu_{static} N$$

When the object is moving, the friction force then becomes a constant. By moving, I mean the object is moving relative to the surface. The kinetic friction is given by:

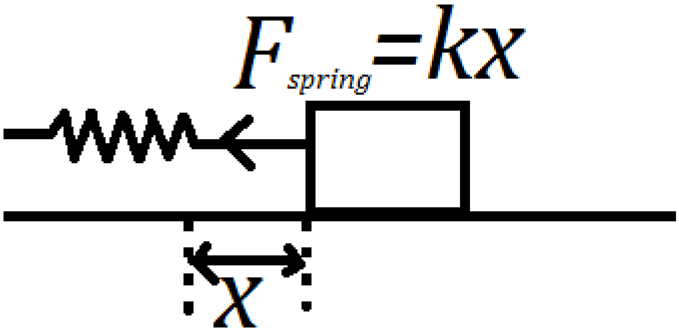
$$F_{kinetic} = \mu_{kinetic} N$$

2.4 Elastic Spring Force

As the name suggests, this is the force exerted by a spring. The law that governs this force is Hooke's Law:

The force required to stretch or compress a spring is directly proportional to the extension of the spring, assuming the elastic limit has not been reached.

$$F_{spring} = -kx$$



The negative sign in the equation is really because force is a vector. If x is forward, the spring force, F_{spring} will pull it backwards. The idea where the force is directly proportional to a certain displacement has an important role in a future topic on oscillations: Simple Harmonic Motion.

2.5 Tension Force

Tension is just a pulling force exerted by things like ropes, strings(or anything that is "one dimensional"), and acts along the object. A spring force is actually a specific type of tension force. Like a normal force, there isn't really an equation.

2.6 Electric and Magnetic Forces

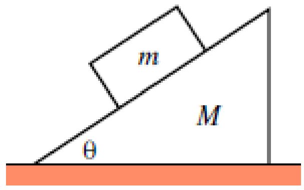
I'll come back to this more specifically in future topics in electromagnetism. This chapter focuses on those forces that are more mechanical nature.

3 The Approach

The types of questions can generally be broken into two types: statics and dynamics. But they are really all about the same equation: $F = ma$. Static is when $a = 0, F = 0$. Dynamic is when $a \neq 0$.

I'll explain the steps using a problem which I wasn't able to do when I was in secondary 3 and 4. But try to follow my thinking process. You can try to draw or write some things when reading this.

Sliding plane (Morin, INTRODUCTORY CLASSICAL MECHANICS) A block of mass m is held motionless on a frictionless plane of mass M and angle of inclination θ. The plane rests on a frictionless horizontal surface. The block is released. What is the horizontal acceleration of the plane?



3.1 Break down the system into components

Typically, a system is made up of multiple components interacting with each other. Analysing it as a whole may be too difficult or it may simply not give us sufficient information to solve the problem.

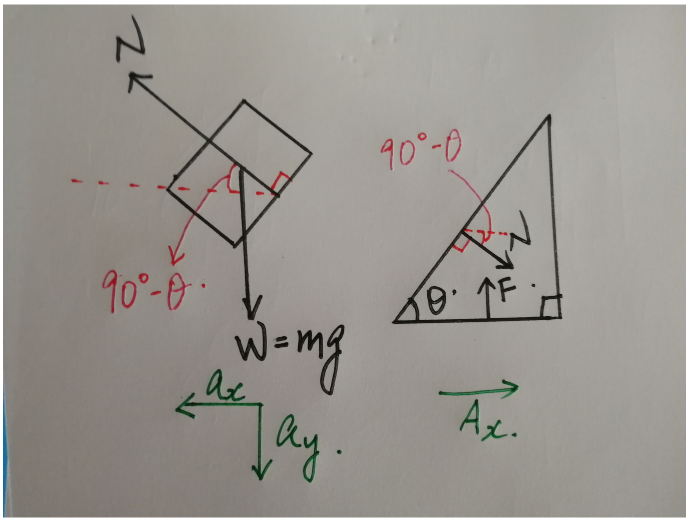
In our problem, it is probably obvious that we should break it down into **the block** and **the plane**. We can include the horizontal surface too but it is quite irrelevant. After all, the floor doesn't really move.

Do note however, that the bodies are still interacting with each other. The forces on each body can still be related in value.

3.2 Draw Free Body Diagram for each component

Take a look at the diagrams I've drawn when explaining the various types of forces. The arrows represent forces on the object. Try to draw exactly where the force acts or where it appears to act(I'll explain why in a future chapter).

Let's do this for our problem. Before we do anything, of course, we need to know what are the forces acting on our bodies. On the block, the forces are the weight(W) and the normal force(N) from the plane below it perpendicular to the plane. On the plane, there are the normal force (that is also N due to Newton's 3rd Law) and the normal force from the horizontal surface(F). (There is also a weight that is not drawn due to a lack of space) Usually there is friction but the question states that every surface is frictionless so let's ignore them in this question. As we know, forces are vectors and their directions are important, so let's take note of the angles.



3.3 Note the constraints

a_x, a_y, A_x are the acceleration of the respective objects as seen in the perspective of a stationary observer. I only gave one variable to define the acceleration of the plane because I know that the plane is restricted to move in a horizontal axis. This means the forces on the plane must work out just nice such that there is only a horizontal component. Basically, the normal force, F, must perfectly cancel out the vertical component of force N as well as the weight(which is not drawn because there is not enough space).

$$F = N\, sin(90^\circ - \theta) + Mg = N\, cos\theta + Mg$$

Another constraint we have is that the block must stay on the plane. Given the initial velocity is zero, the ratio of vertical displacement, velocity and acceleration to that of the horizontal must satisfy the equation.

$$tan\theta = \frac{a_y}{a_x + A_x}$$

3.4 Lay down the equations and solve them

Now, all that is left is to write down the equations and solve them. Forces on the block in the x-direction must give the mass multiplied by the acceleration in the x-direction. Since the weight only has a vertical component, we have:

$$N\, cos(90^\circ - \theta) = N\, sin\theta = ma_x$$

Likewise, repeat for the other axes as well as for the plane.

$$mg - N\, sin(90^\circ - \theta) = mg - N\, cos\theta = ma_y$$

$$N\, cos(90^\circ - \theta) = N\, sin\theta \;= MA_x$$

We have all the equations we know. All that is left is to solve them. Replace $a_y = tan\theta\,(a_x + A_x)$

$$mg - N cos\theta = m(a_x + A_x)tan\theta = m(\frac{N\,sin\theta}{m} + \frac{N\,sin\theta}{M})tan\theta = N(sin\theta + \frac{m}{M}sin\theta)tan\theta$$

$$mg = N(\frac{sin^2\theta}{cos\theta} + \frac{m}{M}\frac{sin^2\theta}{cos\theta} + cos\theta) = N(\frac{sin^2\theta}{cos\theta} + \frac{m}{M}\frac{sin^2\theta}{cos\theta} + \frac{cos^2\theta}{cos\theta}) = N(\frac{M + m\,sin^2\theta}{M\,cos\theta})$$

$$N = \frac{Mmg\,cos\theta}{M + m\,sin^2\theta}$$

The questions asks for A_x.

$$A_x = \frac{N\,sin\theta}{M} = \frac{mg\,cos\theta\,sin\theta}{M + m\,sin^2\theta}$$

The answer the book gives is $\frac{mg\,tan\theta}{M(1+tan^2\theta)+m\,tan^2\theta}$. But I can show that it is the same as my answer.

$$sin^2\theta + cos^2\theta = 1$$

$$tan^2\theta + 1 = \frac{1}{cos^2\theta} = sec^2\theta$$

$$\frac{mg\,tan\theta}{M(1 + tan^2\theta) + m\,tan^2\theta} = \frac{mg\,tan\theta}{M\,sec^2\theta + m\,tan^2\theta}$$

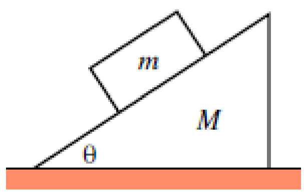
Multiply both top and bottom by $cos^2\theta$.

$$\frac{mg\,tan\theta}{M\,sec^2\theta + m\,tan^2\theta} = \frac{mg\,cos\theta\,sin\theta}{M + m\,sin^2\theta}$$

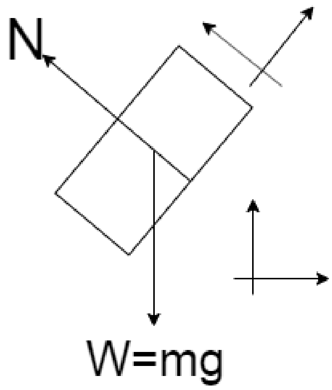
The math looks really *disgusting* but you will get used to it when you are more familiar with both the Physics and the Math.

4 Alternative Coordinate Systems

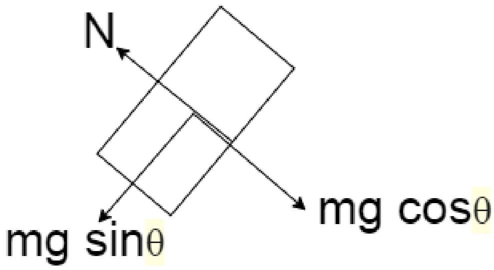
Honestly, who said that we need to stick to left/right, up/down as our coordinate system? In fact, we don't. The normal Cartesian coordinate system is just more intuitive. Sometimes, it is simpler to use other coordinate systems.



Consider a plane that is stuck to the ground and a block is allowed to slide on it. For now, let's assume there is no friction. Let's go through the steps. Our system in this case is just the block since the plane is not moving. Draw a free body diagram:



Since the block moves along the plane, it is probably easiest to use a coordinate system with respect to the plane: perpendicular and parallel to the plane. Resolving the weight to along those two axes and we get:



From this, we can know that

$$N = mg\ cos\theta$$

or else the block will sink into the incline.

Also, the block will accelerate along the incline with an acceleration of $gsin\theta$, assuming there is no friction.

This can be useful in solving various problem. Just a question to ponder: is the normal force and the acceleration along the incline the same as for the question on sliding plane.

Just to summarize, in this chapter, we talked about the various types of forces and how to solve problems by combining these with Newton's Laws.