

Forces: Newton's Laws

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1 Introduction

We all have a certain sense of what forces are - push or pull. What this tells us is that force has a direction and a magnitude, therefore, it is a vector quantity.

It is not sufficient to merely know how does an object move, but also what causes and affects motion. In this chapter, we will see forces are what changes an object's motion.

2 Newton's Laws of Motion

Laws are the fundamental rules that tells us what happens in our physical world. The laws should be what guides our thinking, not the other way round. Let's just accept these laws as the absolute truth, because I have no idea how these laws can be proven. They are more or less just experimental facts.

For the sake of solving problems, I believe one skill is important — *imagination*. I don't mean to understand or come up with laws in your own ways. I mean to be able to imagine the system in your head.

Let's first imagine an object floating in free space, no friction, no air resistance, no gravity, **no net force acting on the object**, just floating. Obviously it will not start moving. Now, let's imagine the object as moving initially with velocity vector \vec{v} . What will happen to its motion afterwards?

Apparently, it will remain in the same velocity \vec{v} . It might contradict what we normally see, as all object seems to come to a stop at a certain point. But this is due to the initial assumption(which is bolded above) not being met. In real life, there are always dissipative forces such as friction. This leads us to the first law.

2.1 Newton's 1st Law of Motion

Given **no net external force**, an object **at rest stays at rest**, and an object **in motion stays in motion** with the **same speed** and in the **same direction**.

We often call this inertia: the tendency for an object to remain in uniform motion. This law can be simple but it has a significant meaning. It defines what is known as an **inertial frame**. It is basically when you don't push an object, the object will not start moving by itself. Can you imagine this happening? You start flying upwards when there is absolutely no force acting on you. Probably not since when you are standing still on Earth, it gives you an extremely good(not perfect) inertial frame of reference. If an object can start accelerating without any external force, then there is no purpose in us discussing forces in the first place.

F1. Imagine a cart moving with constant velocity, which has a cannon on it that can shoot a ball upwards. Will the ball land on the floor at the position where it was shot, or will the ball land back in the cart? (Assuming no air resistance)

This video gives the answer. Not exactly a cart but the idea is exactly the same. <https://www.youtube.com/watch?v=Qvep9kWblfs>

Well, in your frame(not moving with the cart), the ball initially has a horizontal velocity. Given the force is only upwards, according to Newton's 1st Law of Motion, its horizontal motion should not be affected.

Now, what if there is an external force? In fact, a net external force. From the first law, we know that if force, $F = 0$, velocity is constant, that means acceleration $a = 0$. So, probably what a force really does is to create an acceleration. Apparently, F is directly proportional to a , and the constant of proportionality is what we usually call the mass, m .

Here, I'll bring in a new quantity called momentum, p . Mathematically, momentum of an object is the product of its mass and velocity(units $kg\ m\ s^{-1}$).

$$p = mv$$

We need this quantity to define Newton's 2nd Law of Motion.

2.2 Newton's 2nd Law of Motion

The rate of change of momentum of a body is directly proportional to the resultant (net) force acting on the body and in the direction of the resultant force.

$$F = \frac{dp}{dt} = \frac{d}{dt}mv$$

Or if you are not comfortable with calculus.

$$F = \frac{\Delta p}{\Delta t} = \frac{\Delta mv}{\Delta t}$$

Usually, mass is constant. This then becomes something that might be more familiar to most people.

$$F = m \frac{dv}{dt} = ma \quad (\text{units : } kg \, m \, s^{-2} = N)$$

N (Newton) is the units for force.

F2. In the previous chapter, we have talked about centripetal acceleration. Consider a rock tied to the end of a rope and swung in a circle of radius r at a constant velocity v . What is the tension force in the rope?

The centripetal acceleration as we know is the net acceleration of an object undergoing uniform circular motion.

$$a = \frac{v^2}{r}$$

$$F = ma = m \frac{v^2}{r}$$

Simple as that. Does it sound intuitive? The faster you swing a rock, the tauter the rope gets.

Imagine you are in a deep swimming pool with your friend. Your body is not touching any floor or wall. You push your friend away. Do you remain stationary or will you also start moving? Most likely you will start moving. This is the whole idea of Newton's Third Law of Motion.

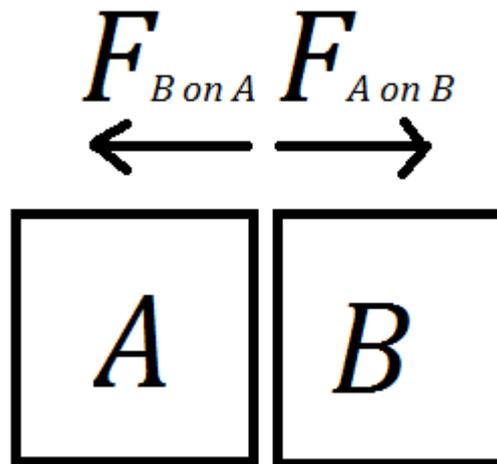
2.3 Newton's 3rd Law of Motion

If A exerts a force on B, B will also exert a force on A that is equal in magnitude but opposite in direction.

Rockets work using this law. A rocket can move upward because it pushes hot air downwards. This means the air also exerts an action-reaction force on the rocket, causing it to move upwards.

F3. Consider two masses pushing against each other. What is the change in total momentum of the system? The total momentum is equal to the vector sum of the momentum of each mass.

Let's use rightwards as positive and leftwards as negative, since momentum is a vector and direction does matter.



As we know from Newton's 3rd Law of Motion, $F_{B \text{ on } A} = F_{A \text{ on } B}$. How to relate force to momentum? We need to use Newton's 2nd Law of motion.

$$\Delta p_A = -F_{B \text{ on } A} \Delta t$$

$$\Delta p_B = F_{A \text{ on } B} \Delta t$$

The negative sign in the first equation is due to the direction, and it is obvious that the force is exerted for the same amount of time, Δt .

With all these information, we know that:

$$\Delta p_A + \Delta p_B = 0$$

The total momentum does not change. This is an important result derived from Newton's Laws, known as the Principle of Conservation of Linear Momentum. We'll come back to this in the future.

Just to summarise, this chapter talks about the 3 Newton's Laws of Motion. Just to recall, the first is the Law of Inertia, the second links force and acceleration, while the third is about action and reaction.

This may be a little heavy. However, these are necessary knowledge before I go on to anything else.

Go take a look at SJPO General round 2016 Q1 and Q2.