Vectors

1 When 1+1=$\sqrt{2}$

This is one of the first few topics taught in Physics. Vectors are quantities with both a direction and a magnitude. Instead of just asking how far have I gone, how fast am I going, I also have to ask where have I gone, where am I going. .

If you step 1 metre forward, and 1 metre leftward, you have moved a distance of 2 metres. Distance is not a vector quantity. Overall, you have moved a displacement of $\sqrt{2}$ metres in the front-left direction. Displacement is a vector quantity.

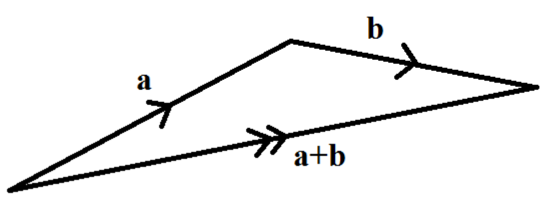
2 Representations

Most of the times, a vector is represented by an arrow. Its length represents the magnitude, and the direction which the arrow is pointing represents the direction.

$\vec{v}$, $\mathbf{v}$ are different ways to write a vector. It is hard to bold an alphabet when writing, so sometimes people put a curly line below their alphabet. When representing a vector going from point A to point B, we write $\vec{AB}$. Vectors with a magnitude of 1 are called unit vectors. They are written like $\hat{v}$.

3 Vector Addition/Subtraction

1. When 2 vectors add, as direction is involved, we cannot simply add up the magnitude. We have to connect the 2 arrows.



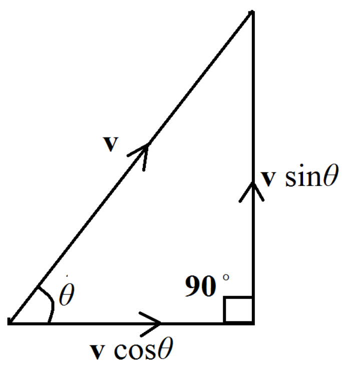
2. Sometimes, we need to do some parallel shifting. A parallel shifting does not affect the direction of a vector, therefore the vector is still the same vector.

3. The negative of a vector is just a vector of the same magnitude/length in the opposite direction.

4 Vector Resolution

This is the most important property of a vector for us. It can be broken down into perpendicular axes. This is similar to Cartesian coordinates in coordinate geometry. A point *(a,b)* represents a point that is a units in the x-direction and b units in the y-direction from the origin. As we can see, the x and y axis are perpendicular to each other.

By convention, people write $\hat{i}$,$\hat{j}$, $\hat{k}$ as a unit vector (i.e. magnitude of 1) in the x,y,z directions respectively.



In the above diagram, all the values are referring to the magnitude only, the directions are given by the arrow. I used some trigonometry (SOHCAHTOA) in getting the values. Using what I said in the previous paragraph, I get:

$$v\ cos\theta\ \hat{i} + v\ sin\theta\ \hat{j} = \vec{v}$$

Sometimes, people also write like this.

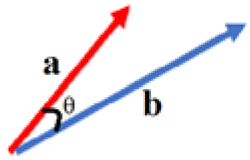
$$\vec{v} = \begin{pmatrix} v\ cos\theta \\ v\ sin\theta \end{pmatrix}$$

Anyways, when two vectors add, they give another vector. By doing vector resolution, it helps break a vector into separate components do we can deal with them separately. This is because x "does not affect" y.

The next part is about some Math that people do with vectors. Not in High School Math syllabus (I think). Not sure if tested in SJPO, but it does help in understanding future topics. I'll skip the detailed Math parts.

5 Dot Product

I like to think of dot product as how much two vectors overlap. I'll do a vector resolution of one of the vectors and find the component which it is parallel to the other vector.



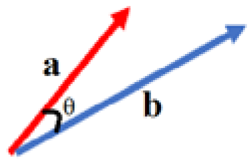
For vector **a**, $a\ cos\theta$ is parallel to **b**, $a\ sin\theta$ is perpendicular to it. $a\ cos\theta$ is called the length of projection of **a** on **b**. Vice versa for **b** on **a**. The dot product is given by:

$$\mathbf{a} \cdot \mathbf{b} = \mathbf{b} \cdot \mathbf{a} = ab\ cos\theta$$

It gives us a scalar.

6 Cross Product

Cross product on the other hand gives a vector, which is perpendicular to the two original vectors. I'll use the same diagram as before.



How to compute a $\times$ **b** :

Take out your **left** hand. Point your fingers in the direction of **a**. Somehow rotate your wrist such that your fingers now point in the direction of **b**, note the angle swept out by your fingers. The magnitude of **a** $\times$ **b** is given by:

$$\mid \mathbf{a} \times \mathbf{b} \mid = ab\ sin\theta$$

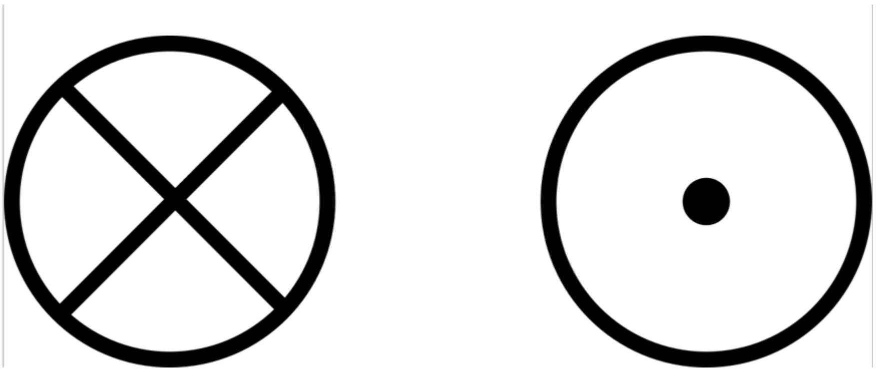
Then what about its direction? It is in the direction of the thumb when you are rotating your wrist. You may ask which angle to use, θ or 360°-θ, but the truth is it doesn't matter. If the angle swept out is θ, your thumb should be into the paper. If the angle swept out is 360°-θ, your thumb should be pointing out of the paper. Since sin(360°-θ)=-sinθ, the direction of the resultant vector should be the same regardless.

Also,

$$\mathbf{a} \times \mathbf{b} = -\mathbf{b} \times \mathbf{a}$$

7 Into and Out of the Paper

In Physics, a lot of things are bound to be in 3D, but a paper is in 2D. Therefore, there is a way to represent things coming out and going into the paper.



The one on the left, refers to something going into the page, while the one on the right refers to something coming out of the page. This something can be arbitrary.

Think of it as an arrow. At the front, there is the pointed part and at the back, there is the feathery part. When the arrow is coming at you, you will be seeing the pointed part, just like the dot on the right pointing out of the paper. Likewise, when the arrow is going away from you, you will be seeing the cross.

This is only a small proportion of what you will learn for vectors in the future. There is a way of calculating dot product and cross product without knowing the angle between two vectors. You can read up more about it.