Calculus

1 Before we start......

Calculus is a Secondary 4 A. Math topic if I remember correctly. It is not explicitly stated that it will be required in SJPO, but I do see certain questions that require some simple calculus. Sometimes they even gave the formula for you to do the calculus. Nonetheless, I am sure the understanding of it is useful in future learning.

2 Circle: Zero or Infinite Edges

Calculus is the study of continuous changes. Looking at a circle, we may think of it as having no edges(that's what I was told in Kindergarten). But when learning calculus, we start zooming in to the edge of a circle, zooming in so much that the edge looks like a straight line.

In calculus, we deal with infinitesimal changes, changes that are very very very small.

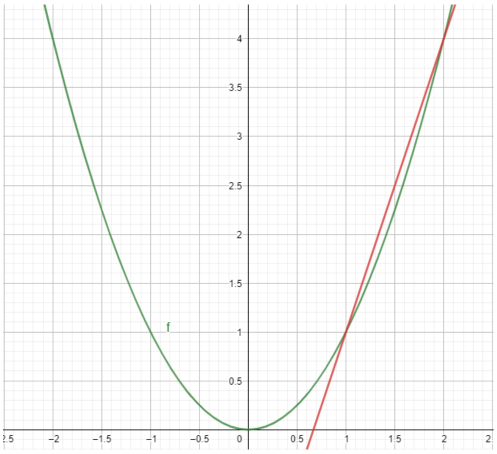
3 Derivatives: Gradient of a Curve

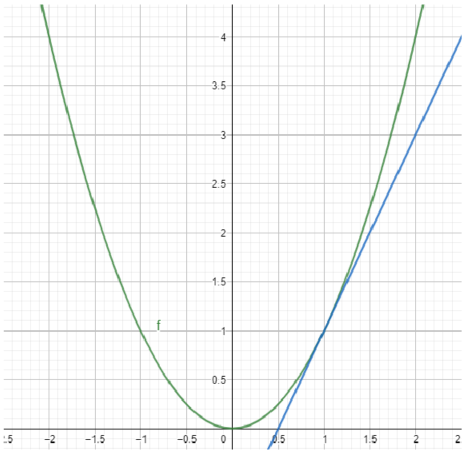
In coordinate geometry, we have learnt that the gradient of a linear graph is $\frac{\Delta y}{\Delta x}$ or rise over run.

However, what about the gradient of a curve? Probably we can guess that the gradient at every point is different and it is continuously changing.

We will have to zoom in to only one point of a curve. That is when Δy and Δx approaches zero (yeah this may be a little hard to grasp), and we get a line that is tangent to the curve. Then, we calculate the gradient of that tangent.

I'll illustrate this idea using two diagrams in the next page.





Both graphs are graphs of $y = x^2$. Both graphs also have another straight line passing through the coordinate *(1,1)*. On the graph on the left, the straight line also intersects the graph at another point. If we push this another point of intersection closer to (1,1), so close that it is virtually right on (1,1), the straight line then becomes a tangent to the graph, like the one shown on the right. If we calculate the gradient of that tangent, we get a gradient of 2.

Time to restate everything I just said using Math. As you can see from the equation below, it looks just like the rise over run, $\frac{\Delta y}{\Delta x}$ equation. The limit as Δx approaches zero:

$$\lim_{\Delta x \to 0} \frac{(x + \Delta x)^2 - x^2}{\Delta x} = \lim_{\Delta x \to 0} \frac{(x + \Delta x - x)(x + \Delta x + x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{\Delta x(2x + \Delta x)}{\Delta x}$$

The idea of limits is where a variable reaches very close, but not yet, to a certain value. So long as Δx is not zero, I can just cancel it off from the top and bottom, leaving me with just $2x + \Delta x$. But as Δx approaches zero, it becomes insignificant compared to $2x$, so it might as well doesn't exist. After all this we get $2x$.

If we substitute with $x = 1$ for the point of the graph which the line intersects, we get a value of 2, which is exactly the gradient of the tangent that we got.

The commonly-accepted ways of writing the derivatives are like this:

$$f(x) = y = x^2$$

$$f'(x) = \frac{dy}{dx} = \frac{d}{dx}y = 2x$$

Your Math teachers may get annoyed when I write this because $\frac{d}{dx}$ is a notation. However, it is really convenient to think of it as a fraction. dy and dx are just when Δy and Δx approaches zero respectively.

In fact, I can extend from $y = x^2$ to get

$$\frac{d}{dx}x^n = nx^{n-1}$$

4 Integration: Area below a Curve

Integration is literally the reverse of finding a derivative. Let me explain what I mean. We claimed that derivatives are the gradient of a tangent. In other words, a very small change in y (so small that it is close to zero), δy, can be approximated by:

$$\delta y \approx \frac{dy}{dx}\delta x$$

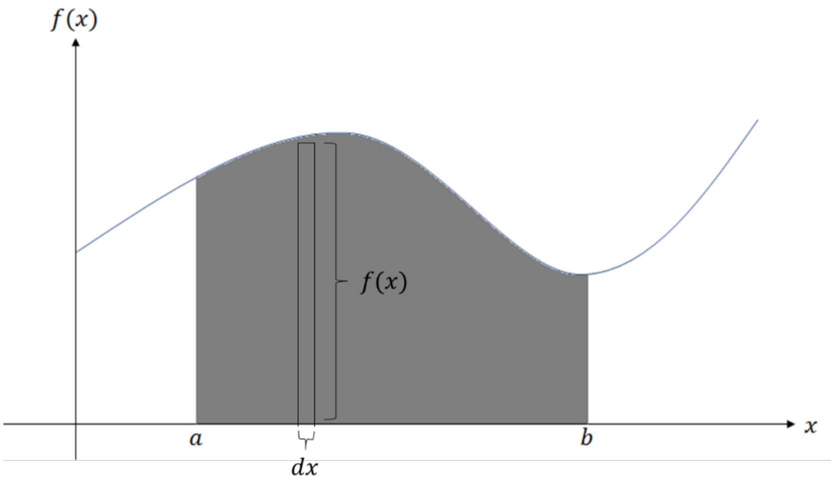
The act of integration, is to add up all these individual δy, and this should give me the total change in y (Δy). In Mathematical notation:

$$\delta y_1 + \delta y_2 + ... + \delta y_n = \sum_{i=1}^{n} \delta y_i \approx \Delta y$$

Σ is just another way of writing summation. The summation should get closer and closer to the exact value of Δy as δy approaches zero, and I have an infinite number of these δy. In Mathematical notation:

$$\int dy = \int \frac{dy}{dx} dx = y + C$$

The $+C$ is important, because I am not talking about the exact value of y. I am talking about a change in y. C is just an arbitrary constant, dependent on the initial conditions in the situation.



Another way to look at integration is that it is finding the area below a certain curve. Looking at the diagram above, each strip has an area $f(x)\,dx$. To find the whole area, do an integration: $\int_a^b f(x)\,dx$.